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# Joint Energy Capacity and Production Planning Optimization in Flow-Shop Systems

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## Abstract

This study introduces new probabilistic constraints and objective functions to manage the uncertain nature of the renewable energy sources in single-item capacitated lot sizing problem for flow-shop configurations by integrating the capacity contract selection problem with multiple energy sources. The aim of the probabilistic models built by considering the different probabilistic constraints and objective functions is to provide a decision-making tool and to promote the use of renewable energy sources in manufacturing industry despite of their stochastic nature. Mixed Integer Non-Linear Programming models are proposed by integrating the uncertainty of the renewable energy sources based on different features. The developed models are tested on a small-size instance and the results of the models are compared in terms of economical, ecological and reliability aspects.

*Keywords:* Lot Sizing, Flow-Shop, Renewable, Probabilistic, Chance-Constraints, MINLP

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## 1. Introduction

According to the report of U.S. Energy Information Administration, world energy consumption will grow by nearly 50 % between 2018 and 2050 [1]. Another noteworthy detail in the same report is that world's energy demand still

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mainly depends on fossil fuels which are responsible for the increasing carbon emissions. The same report reveals that the industrial sector, which includes mining, manufacturing, agriculture, and construction, accounts for the largest share of energy consumption of any end-use sector, accounting for more than 50% over the entire projection period [1]. A recently published report by the Carbon Disclosure Project (CDP), reveals that between 1988 and 2015, 100 of all the hundreds of thousands of companies in the world have been responsible for 71 % of the global GHG emissions [2]. All these facts make the industrial sector the hottest target to work on to reduce the high carbon emission levels. It is a well-known fact that energy generation from renewable energy sources is the only way to manage the energy starvation of the nations and to protect the planet as well. If the conventional energy sources supplied to the industrial customers are diversified with the renewable ones; while the energy reserves of the world can be used more efficiently, environmental targets can be achieved significantly by declining carbon emission levels.

Despite all these positive outcomes of renewable energy sources, the transition of the renewable sources in the industrial sector shows a slow progress. Even though there are several socio-economic and technological barriers that hinder the deployment of the renewable energy sources, their intermittent and stochastic nature can be attributed as one of the main reasons that discourage the industrial customers who need the continuity of the energy supply to sustain the production activities without any disruption. At this point, the key question that comes to the minds of decision makers is when and how much product must be produced to meet the demands of the customers, and how this plan can be carried out more safely by considering the random nature of renewable energy sources. These two questions can be merged and translated as *lot sizing problem under renewable energy sources*.

The main objective of the lot sizing problem is to determine the size of the production and inventory lots by minimizing production, set-up and holding related costs and satisfying the demand of the number of products over a defined

time horizon. The viability of the decisions made at a tactical level is heavily depending on the success of the implementation of them at the operational level.

Based on this fact, several studies which combine lot sizing and scheduling problems have been produced by researchers over the years. In particular, since the lot sizing problem for flow-shop systems is studied in this paper, it would be useful to mention the previous studies on this type of system. Mohammadi et al. [3] studied the multi-level multi-product capacitated lot sizing problem for pure flow-shop system. Babaei et al. [4] handled multi-level and multi-period capacitated lot sizing and scheduling problem with sequence-dependent set-ups and set-up carry over in flow-shop environment. Ramezani et al. [5] considered the problem of lot sizing and scheduling of multiple product types in a capacitated flow-shop with machine availability constraints for multi-period planning horizon and three MIP-based heuristics based on iterative procedures are used to solve problem instances. Several works [6, 7, 8, 9, 10] developed mathematical models and heuristic approaches for the integration of lot sizing and flow-shop scheduling with lot streaming. Gong et al. [7] and Han et al. [8] proposed to consider blocking constraints for the lot-streaming problem in flow shops systems. Han et al. [9, 10] studied the same problem with interval processing time and machine breakdowns. Ramezani and Saidi-Mehrabad [11] addressed lot sizing and scheduling problem of a flow-shop system with capacity constraints, sequence-dependent setups, uncertain processing times and uncertain multi-product and multi-period demand. The uncertain parameters are modeled through probability distributions and chance-constrained programming theory. Mahdih [12] studied the integrated capacitated lot sizing and scheduling problems in a flexible flow line system where flow-shop and parallel shop manufacturing systems are hybridized. Villadiego et al. [13] studied integrated lot sizing and sequencing problem in a permutation flow-shop with machine sequence-dependent setups and proposed an iterated greedy heuristic to cope with the complexity of the problem. Torkaman et al. [14] studied multi-product multi-period capacitated lot sizing problem with remanufacturing for flow-shop systems by including sequence-dependent setups and set-up carry over.

Due to the rising awareness for constructing environmental friendly production systems, lot sizing problem has been handled by controlling carbon emission levels. Several works [15, 16, 17, 18, 19] can be exemplified for the studies based on the environmental friendly lot sizing problems. The main characteristic of these studies is that the energy aspect is integrated into the different variants of the lot sizing problems by targeting to limit the carbon emission levels in accordance with the different carbon regulations and to minimize the operational costs respecting the ecological ones. It can be said that the starting point of the previous studies is ecological concerns rather than economical ones.

Different from referenced environmental-based lot sizing studies, Masmoudi et al. [20] considered the energy as a resource whose cost is required to be considered. They took into account the power and electricity consumption costs and proposed a model for capacitated single-item lot sizing problem for flow-shop production systems. In their study, the total power demand is calculated for each period and limited by the maximum amount of power which can be supplied by the energy supplier. The developed model aims to identify the optimum production quantities by minimizing the production and energy costs. To the best of our knowledge, the study of [20] is the first attempt that combines lot sizing problems in flow-shop system with energy consideration.

The main shortcoming of the model proposed by [20] is that since the energy consumption pattern is formed according to the varying energy prices, the power demand of the system fluctuates considerably. However, in real life, energy suppliers can provide the energy to their customers in a certain balance and, to keep this balanced energy flow, they negotiate with the customers for different power options including different tariffs and reach an agreement. Inspired from the real life practices, Rodoplu et al. [21] extended the model proposed by [20] to the optimum energy contract capacity selection problem. The proposed model in the study of [21] aims to identify the optimum production quantity and to generate the optimum energy mix. This is done while taking into account

the demand, the system and energy related constraints, the power need of the production system and the capacity options offered by the supplier. To the best of our knowledge, the presented study in [21] is the first attempt that combines optimal energy contract selection problem with lot sizing problem for flow-shop configuration by integrating renewable energy sources. Rodoplu et al. [22] improved the previously presented mathematical model in [21] in terms of computing the exact value of the peak power demand and implemented Fix-and-Relax heuristic to solve the problem efficiently. Therefore, the studies of [21, 22] provided more realistic decision-making tool to the decision makers in the manufacturing industry since their studies consider production planning and energy purchasing constraints and objectives simultaneously (Table 1).

Nonetheless, there is still room for improvement. The common point of these two works is that the single-item lot sizing problem is handled by assuming that the renewable energy resources will be uninterruptedly supplied to the consumer as agreed in the contract. However, these weather-dependent energy sources are risky due to their intermittent nature and they might not be fully met as they are agreed in the contract. The customers, who prefer to mix non-renewable and renewable energy sources to achieve environmental targets, must consider this risk. Therefore, to provide a better decision-making tool to the decision makers, it is necessary to consider this unavailability risk in the supply of renewable energy sources during the production planning and energy purchasing process. Based on this motivation, Rodoplu et al. [23] proposed a preliminary model in order to identify the optimum quantity for production lots in a flow-shop system and to generate the optimum energy mix which can cover the need of the system by considering the stochastic nature of the renewable energy sources. The chance-constraint modelling approach is applied to integrate the risk level in the supply of the renewable sources into the lot sizing problem.

Different from the study of Rodoplu et al. [23], in which the proposed constraints are displayed in brackets in Table 1, this paper proposes two types of probabilistic objective functions and introduces three new types of probabilistic

constraints as summarized in Table 1 which highlights the contributions of this work compared to existing ones. With this attempt, it is aimed to provide alternatives to evaluate the expected cost of the disturbance in the supply of the renewable energy sources. The newly built models can give an insight to industrial customers about the cost that might need to bear in case of failure in the supply of the renewable energy sources. The provided decision-making tool allows the customers to organize their production in a safer way under the uncertain availability of the renewable energy sources.

Features	[20]	[21, 22]	[23]	This work
Energy cost consideration	✓	✓	✓(10)	✓
Contract capacity selection		✓	✓	✓
Use of traditional energy	✓	✓	✓	✓
Use of renewable energy		✓	✓	✓
Probabilistic objective function(s)				✓
Service level constraint.(required ener.)			✓(3, 14,15)	
Service level constraint(contracting ener.)				✓
Probabilistic constraint(average power)				✓
Probabilistic constraint(average failure)				✓

Table 1: A comparison of this work with the previous studies

The remaining of the paper is organized as follows: In Section 2, the handled problem is stated explicitly and the assumptions are detailed. In Section 3, the newly proposed probabilistic constraints are classified and explained in a general frame. In Section 4, the developed probabilistic objective functions are introduced. In Section 5, developed constraints and objective functions are studied with the consideration of exponential distribution. In Section 6, a numerical study is conducted to underline the impact of the energy constraints on the production planning and to illustrate the effects of the different types of constraints on the obtained solution. In Section 7, an instance inspired from an industrial case is studied. Section 8 concludes this study with some remarks

and perspectives.

## 2. Single-Item Lot Sizing Problem with Energy Contract Selection

### 2.1. Problem Statement & Objective

The considered production system is composed of  $N$  machines and  $N-1$  buffers (Figure 1). The planning horizon is separated into  $T$  periods. Each period ( $t=1\dots T$ ) is depicted with its length ( $L_t$ ), the deterministically known demand ( $d_t$ ) to be met, and electricity prices ( $Co_t$ ).

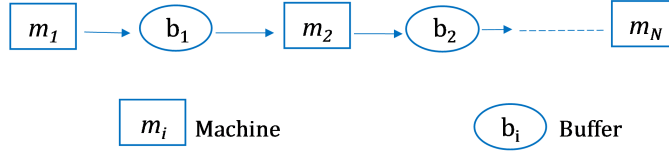


Figure 1: A typical manufacturing system with  $N$  machines and  $N-1$  buffer [24]

The purpose of the developed model is to determine the optimum production quantities ( $x_{m,t}$ ) on each machine ( $m=1,\dots,N$ ) and in each period ( $t=1,\dots,T$ ) by minimizing the costs related to production, holding, set-up and energy and to identify the required maximum power demand in each period by considering the risks which can occur in the supply of contracted renewable energy sources.

### 2.2. Assumptions

The assumptions related to the handled problem are categorized into two groups: Production Related Assumptions and Energy Related Assumptions.

The primary production related assumptions can be listed as follows: The external demand is dynamic and deterministic and must be satisfied at the end of each period. The backlogging is not taken into account. For each machine, a single set-up is allowed. The first machine is never starved and the last machine is never blocked at each period. A machine  $m$  can only start the production



when all the quantity to be produced ( $x_{m,t}$ ) is available at the output of the previous machine. Therefore, vertical interaction is considered as in the studies of [25] and [22].

Energy related constraints can be ordered as follows: The use of  $K$  types of energy sources (traditional, wind, solar) is assumed. The on-site generation is not considered in this study, therefore, since the considered conventional and renewable types of energy sources are supplied by the energy supplier, the unit price of energy and the tariffs are fixed by the suppliers. A given number  $R_k$  of capacity options (which are not necessarily equal for all sources) can be offered by the suppliers for each type of energy sources  $k=1, \dots, K$ . The  $R_k$  proposed options form the energy contract vectors  $V_k = V_{k,1}, \dots, V_{k,R_k}$  for each type of energy sources. Each energy option ( $l=1, \dots, R_k$ ) for each energy source ( $k=1, \dots, K$ ) is portrayed by its subscription fee ( $V_{cost_{k,l}}$ ). The customers are bounded to generate their energy mix by choosing one capacity option from each type of energy source. Contract switching during the planning horizon is not permitted.

The parameters and the decision variables used in the developed models are as follows:

$N$  : Number of machines

$T$  : Number of production periods

$K$  : Number of energy sources

$R_k$  : Number of contract options for energy source of  $k=1, \dots, K$

$V_{k,l}$  : Contract option  $l=1, \dots, R_k$  for energy source of  $k=1, \dots, K$

$V_{cost_{k,l}}$  : Subscription fee for option  $l=1, \dots, R_k$  of energy source  $k=1, \dots, K$

$SL$  : Minimum service level defined by industrial customer

$\alpha_t$  : The decision variable which defines the total power need of each period

$X_s$  : Random variable of solar energy source

$X_w$  : Random variable of wind energy source

$P_{k,l}$  : A binary variable, equal to 1 if option  $l$  ( $l=1, \dots, R_k$ ) is chosen for energy source  $k$  ( $k=1, \dots, K$ ), otherwise 0

$E[X_s]$  : Expected value for solar power generation

$E[X_w]$  : Expected value for wind power generation

$\overline{\nabla}_s$ : Average failure amount based on the contracted power for the solar energy

$\overline{\nabla}_w$ : Average failure amount based on the contracted power for the wind energy

$\gamma$ : Penalty cost

### 3. Developed Probabilistic Constraints

In this section, three new types of probabilistic constraints are proposed to model the stochastic nature of the availability of the renewable energy sources:

- Service Level Based Chance Constraints based on Contracted Renewable Energy
- Average Power Amount Based Probabilistic Constraints
- Average Failure Amount Based Probabilistic Constraints

In the first group of constraints, the chance constraint programming approach is used and the risk in the supply of renewable energy sources is transformed into the risk in the realization of the production plan by introducing *service level* of industrial customers. In [23], the probability of meeting the required renewable energy source is investigated and compared to the service level. Probabilistic constraints named *CS1* and *CS2* are proposed. In this study, the probability of having at least the amount of renewable energy agreed in the contract is compared with the promised service level.

In the second group of constraints, the randomness in the availability of the renewable energy sources is reflected in the model as the expected value of the availability.

Lastly, in the third group of constraints, the expected failure amount that might not be met according to the contracted option is calculated and optimum contract options are selected by considering risky amount of energy.

### 3.1. Service Level Based Chance Constraints

Chance-constraint programming which is also known as probabilistic programming was firstly introduced by [26] to deal with the linear models including uncertain features. Since then, it has been extensively used for the solution of several types of problems such as capacitated facility location [27], supplier selection [28], blood inventory management [29], disassembly scheduling [30], shelter site location [31], vehicle routing problem [32], single-item capacitated lot sizing problem [33].

In particular, the studies of [34], [35], [36], [37] can be referred as the examples for the application of chance-constraint programming for energy management problems. [38] reviews the chance-constraint programming approach with applications in energy management in their study.

#### 3.1.1. Contracted Renewable Energy Based Constraints

In service level based chance constraints; a *service level (SL)*, corresponding to a production rate under the uncertainty of renewable energy sources, is defined by the manufacturer. The *service level (SL)* represents the minimum probability for the availability of renewable energy source. In other words, the probability of available renewable energy amount ( $X_s$ ) is greater than the contracted amount of renewable energy must be at least as much as the specified service level ( $SL$ ) to satisfy the targeted service level. This idea is translated to the following chance constraint:

$$\text{Prob}(X_s \geq \text{the contracted quantity for renewable energy source}) \geq SL \quad (1)$$

This primitive version can be expanded according to the different energy mixes. The proposed constraint in (1) can be expressed **for the traditional and one/two type(s) of renewable energy mix, where  $s$  and  $w$  mean solar and wind energy sources respectively, as follows:**

$$CS3 : (1) \iff P\left(\sum_{o \in \{s,w\}} X_o < \left(\sum_{o \in \{s,w\}}^{R_o} (V_{o,t} \cdot P_{o,t})\right)\right) \leq 1 - SL \quad (2)$$

where:

$\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})$ , is the contracted capacity option for solar energy source  
 $\sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})$ , is the contracted capacity option for wind energy source

At this point, it is necessary to recall the constraints proposed in [23] and emphasize the difference between the two different types of service level based chance constraints. The proposed “required renewable energy” based constraints, named CS1 and CS2, in [23] are as follows:

$$CS1 \& CS2 : P\left(\sum_{o \in \{s,w\}} X_o < \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l})\right) \leq 1 - SL \quad \forall t = 1, \dots, T \quad (3)$$

The main difference between the constraints (2) and (3) is that periodical power need ( $\alpha_t$ ) is taken into account in one but not in the other. In the constraints (2), the service level is satisfied according to the probability of having at least contracted amount of renewable energy. The constraints (2) bring more rigidity to the model, thus, keep the industrial customer on the safer side. The difference between the constraints (2) and (3) will be highlighted with a numerical study in Section 6.

Therefore, the “contracted renewable energy” based chance constraints are developed in a global manner. In the following section, the second group of constraints which is defined as “*Average Power Amount Based Probabilistic Constraints*” is explained broadly.

### 3.2. Average Power Amount Based Probabilistic Constraints

The power generation from solar and wind energy sources depends on the solar irradiation and wind speed, respectively. When the solar power generation is considered, the irradiation value changes continuously from the first hours of the sunrise to the sunset. The same type of fluctuations in solar irradiation can be observed over a season. When it comes to the power generation from the wind energy, it is possible to say that the wind speed diversifies during the day or season. In this constraint, it is aimed to take into account the mean value of the

power generation to cope with the stochastic aspect of renewable energy sources. In probability theory, the mean of a random variable ( $X$ ) is defined as *expected value* and the expected value of a continuous random variable is calculated using the probability density function ( $f(x)$ ) via the following formula:

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx \quad (4)$$

Even though the availability of renewable energy sources is uncertain, it is a fact that when the traditional energy source is mixed with the renewable energy sources, the sum of the deterministically known traditional power and the expected renewable power generation must cover the energy need of the production system. This idea can be translated for **the traditional and one/two type(s) of renewable energy mix, where  $s$  and  $w$  mean solar and wind energy sources respectively** as follows:

$$CS4: \alpha_t \leq \sum_{l=1}^{R_{tr}} (V_{tr,l} P_{tr,l}) + \sum_{o \in \{s,w\}} E[X_o] \quad (5)$$

where:

$\sum_{l=1}^{R_{tr}} (V_{tr,l} P_{tr,l})$ , is the selected capacity option for traditional energy source.

As it is shown in the constraints (5), the randomness of the renewable energy sources can be taken into account in the contract capacity selection procedure by calculating the expected values of the random variables according to the given probability density functions. Therefore, the second group of constraints is completed.

In the subsequent section, the constraints which are built on the expected failure amount in the supply of the renewable energy sources are presented in a general frame as the other constraints.

### 3.3. Average Failure Amount Based Probabilistic Constraints

When the clients negotiate for the certain capacity options for the renewable energy sources, they must consider that the complete amount of the contracted

option might not be supplied due to the random weather conditions. In the case of increasing the contracted value, the risk of dissatisfying the whole contracted amount increases and the risky amount which can not be met increases proportionally. In this constraint, the stochastic aspect of the renewable energy sources is taken into account by figuring out the expected failure amount depending on the contracted option.

In case of using one type of renewable energy, solar for instance, the failure amount ( $\nabla_s$ ) for solar energy can be defined as follows:

$$\nabla_s = \begin{cases} \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - X_s & X_s \leq \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}); \\ 0 & X_s > \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}); \end{cases} \quad (6)$$

where:

$\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})$ , is the contracted amount for solar energy source.

From the definition given in (4) and assuming  $X_s$  has a probability density function  $f_s(x)$  the expected failure amount can be calculated as follows:

$$\bar{\nabla}_s = \int_0^{\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})} (\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - X_s) f_s(x) dx \quad (7)$$

Based on the average failure amount ( $\bar{\nabla}_s$ ) obtained in (7), **the average failure amount constraint can be built for the traditional and one/two type(s) of renewable energy mix, where  $s$  and  $w$  mean *solar* and *wind* energy sources respectively** as follows:

$$CS5 : \alpha_t \leq \sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} \cdot P_{k,l}) - \sum_{o \in \{s,w\}} \bar{\nabla}_o \quad \forall t = 1, \dots, T \quad (8)$$

$$\begin{aligned}
(8) \iff \alpha_t \leq & \sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} \cdot P_{k,l}) - \left[ \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) \int_0^{\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})} f_s(x) dx \right. \\
& - \int_0^{\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})} X_s \cdot f_s(x) dx \left. - \left[ \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l}) \int_0^{\sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})} \right. \right. \\
& \left. \left. f_w(x) dx - \int_0^{\sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})} X_w \cdot f_w(x) dx \right] \right] \quad \forall t = 1, \dots, T
\end{aligned} \tag{9}$$

where:

$\sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} \cdot P_{k,l})$ , is the sum of the contracted power amount.

Thus, three types of probabilistic constraints have been proposed. The developed probabilistic constraints can be integrated into the mathematical model presented in [23]. The probabilistic models obtained by the integration of the developed probabilistic constraints in this section and the objective functions which will be detailed in the next section are given in Table 2 and Table 3.

#### 4. Developed Objective Functions

In [23], the authors do not introduce any stochastic aspect of renewable energy sources in the objective function. They develop the objective function by considering the operational and power cost as follows:

$$\begin{aligned}
Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t} x_{m,t} + h I_{m,t} + w_{m,t} y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (V_{cost_{k,l}} P_{k,l})
\end{aligned} \tag{10}$$

The first part of the objective function minimizes the operational costs (including electricity consumption based production cost, holding and set-up costs). In the second part, the power purchasing cost is minimized. Further detail on the notations of the model can be found in [23]. However, in case of

renewable energy shortage, the industrial customer may encounter some situations such as using extra traditional energy to compensate the unmet renewable energy to sustain the production activities. All these cases increase the costs for the customers and this fact should be integrated into the cost function.

Based on this motivation, in this study, two types of probabilistic objective functions are introduced. In the first type, expected cost depending on the failure probability is taken into consideration and added to the classical production and energy purchasing costs. In the second type of objective function, directly the cost of the amount that is expected to be failed in the supply of the renewable energy sources is considered.

#### 4.1. *OF.1 : Expected Cost of The Probability of Failure*

In order to avoid over consumption of traditional energy, it is considered to penalize the probability of the failure in the supply of the renewable energy sources. Accordingly, for the customers who wish to mix traditional energy with only one/two types of renewable energy source, the following objective function is developed:

$$OF.1 : Minz = \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) + \sum_{t=1}^T \gamma \cdot P(\sum_{o \in \{s,w\}} X_o < \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l})) \quad (11)$$

where:

$\alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l})$ : Required amount for renewable energy source.

In the last part, in case of having less solar energy than the required amount since the customer might need to consume more traditional source than planned to compensate the energy gap, the occurring additional cost is added.

In this study, as an alternative to the “required renewable energy based” service level chance constraints presented in [23], “contracted renewable energy based” chance constraints had been developed in the constraint (2). In the



case of introducing the contracted renewable energy based constraints to the model, the expected cost is computed according to the unmet amount of energy based on the contracted power and the objective functions proposed in (11) is remodelled as follows:

$$\begin{aligned}
OF.1.a : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) + \sum_{t=1}^T \gamma \cdot P \left( \sum_{o \in \{s,w\}} X_o < \sum_{o \in \{s,w\}}^{R_o} (V_{o,l} \cdot P_{o,l}) \right) \quad (12)
\end{aligned}$$

In Section 5, these proposed objective functions will be enhanced by considering exponential distribution.

#### 4.2. OF.2 : *Expected Cost of Failure Amount of Renewable Energy*

It is obvious that as long as the value of contracted option for renewable energy increases, it will be more risky to meet all amount of energy contracted for the supplier and, correspondingly, the average amount of failure increases. The following objective function for the use of one/two type(s) of renewable energy with traditional source is proposed by computing the expected failure amount in the supply of the renewable energy sources as in constraint (9) :

$$\begin{aligned}
OF.2 : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \quad (13) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) + \gamma \cdot \sum_{o \in \{s,w\}} \bar{\nabla}_o
\end{aligned}$$

where:

$\gamma$  : Penalty cost per unit.

As shown in Table 5, the power options that are offered to customers are composed of given values. While the industrial customer negotiates for the renewable energy options, he/she should consider that a portion of the agreed power might not be supplied. The key point is to select the best renewable

option which can cover the need of the system and cause the least expected cost under the given climate conditions. In return, this allow the customers to meet the requirements related to the environmental policies and enable them to communicate with their clients with the notion of “green production”. The objective functions (11)-(13) are developed for this purpose.

Two types of objective functions have, so far, been proposed to consider the cost of the uncertainty of the renewable energy sources. The developed constraints and the objective functions are presented globally without specifying any probability distribution for the random variables.

In the following section, the constructed probabilistic constraints (2), (5), (8), and the objective functions (11)-(13) will be studied by considering exponential distribution. Mathematical models that take into account the stochastic nature of the renewable energy sources for the single-item lot sizing problem will be generated by merging the probabilistic constraints and the newly developed objective functions.

## 5. The Case Study Using the Exponential Distribution

As [39] pointed out in their study, the wind power generation extremely appertains to wind speed and the solar power generation is dependent on the solar radiation and environment temperature. To model the wind energy output, the first task is to generate wind speed samples by Weibull distribution which is a mathematical idealization of the distribution of wind speed over time [40], then these samples are transformed to power output by using “*wind speedpower*” curve.

The same approach is conducted for the solar power generation. In this case, the solar radiation is modelled via Beta distribution [40] or Normal distribution as in the study of [41] and the power generation is computed by using the curve of “*radiation-power*”.

In [23], the authors look for the probability of meeting the required renewable energy source by the energy supplier. To do so, it is required to model

the uncertainty based on the “probability-power demand” curve. As long as the power need increases, the risk of not meeting the required power amount increases (Figure 2). Moreover, in recently published study of [42], they prove that the “probability-power” distribution follows the exponential distribution in the case of wind speed is 2m/s. Based on these considerations:

- It is assumed that both of the random variables follow exponential distribution with the parameters  $\lambda_s$  and  $\lambda_w$  independently.
- Since the expected value of average generation of wind and solar energy sources are generally different, it is assumed that  $\lambda_s \neq \lambda_w$ .
- The values of  $\lambda_s$  and  $\lambda_w$  only depend on the type of the energy and they are constant over the horizon.

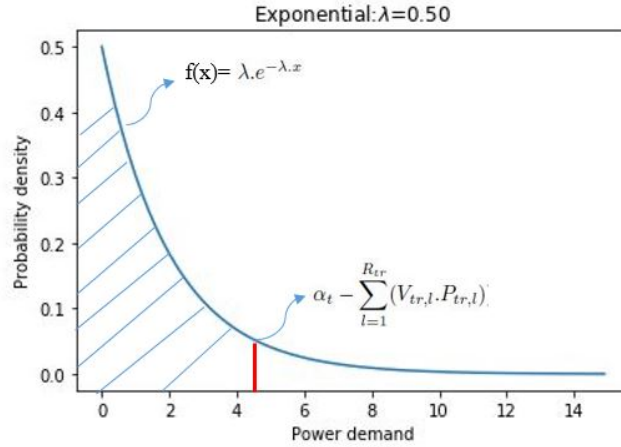


Figure 2: The adaptation of exponential distribution to the power demand

To compute the area under the curve ( $P(X_s \leq \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}))$ ) in Figure 2, a continuous random variable  $X$ , following exponential distribution, with parameter  $\lambda$  is considered. Its probability density function is defined as follows:

$$f(x) = \begin{cases} \lambda \cdot e^{-\lambda \cdot x} & x \geq 0; \\ 0 & x < 0; \end{cases}$$

In this section, the probabilistic constraints and objective functions presented in the previous sections are expanded by considering the exponential distribution.

### 5.1. Probabilistic Constraints Based on Exponential Distribution

#### 5.1.1. Service Level Based Chance Constraints

The constraint CS1 of [23] illustrated in Figure 2 by assuming that the random variable ( $X_s$ ) follows the exponential distribution, with the parameter of  $\lambda_s$ . This constraint is expressed as follows:

$$\sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}) \geq \frac{\ln(SL)}{\lambda_s} + \alpha_t \quad \forall t = 1, \dots, T \quad (14)$$

The constraint CS2 of [23], recalled in (3), is developed as follows:

$$\frac{\lambda_s \cdot (1 - e^{-\lambda_w \cdot Q_t}) - \lambda_w (1 - e^{-\lambda_s \cdot Q_t})}{\lambda_s - \lambda_w} \leq 1 - SL \quad \forall t = 1, \dots, T \quad (15)$$

where:

$$Q_t = (\alpha_t - \sum_{l=1}^{R_{tr}} V_{tr,l} P_{tr,l})$$

The equivalents of contracted renewable energy based probabilistic constraints, which are defined as (CS3) can be obtained in the same way. The constraint CS3, proposed in (2), is transformed as follows by assuming that the continuous random variable follows the exponential distribution:

$$CS3 : \int_0^{\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})} f(x) dx \leq 1 - SL \quad \forall t = 1, \dots, T \quad (16)$$

$$\Leftrightarrow \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) \leq \frac{\ln(SL)}{-\lambda_s} \quad \forall t = 1, \dots, T \quad (17)$$

When two types of renewable energy sources are blended with the traditional source, the left hand side of the constraint involves the sum of the distributions of two independent random variables and to get the probability density function of the sum of the continuous random variables on the left hand side of the inequality, the convolution product is computed as it is done in (15), thus, the equivalence of the constraint (2) with the parameters of the distributions  $\lambda_s$ ,  $\lambda_w$  is obtained as follows:

$$CS3: \frac{\lambda_s(1 - e^{-\lambda_w \cdot Q_t}) - \lambda_w(1 - e^{-\lambda_s Q_t})}{\lambda_s - \lambda_w} \leq 1 - SL \quad \forall t = 1, \dots, T \quad (18)$$

where:

$$Q_t = \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) + \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})$$

To clarify the relation between the randomness of the renewable energy source and the power demand ( $\alpha_t$ ) at period  $t$ , it is necessary to recall the following constraint from [23]:

$$\sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} P_{k,l}) \geq \alpha_t \quad \forall t = 1, \dots, T \quad (19)$$

Constraint (19) implies that the total contracted power must cover the power need of the system. If constraint (17) is employed for a case whose average solar power generation is quite high, (correspondingly, the value of  $\lambda_s$  is quite small) the right hand side of the constraint (17) increases and this case allows the customer to purchase more solar energy. On the other hand, if the price of the traditional energy source is affordable, more traditional can be contracted and the abundance of energy can enable many machines work in parallel since ( $\alpha_t$ ) is satisfied easily.

### 5.1.2. Average Power Amount Based Probabilistic Constraints

Considering a random variable  $X_s$  following the exponential distribution with the parameter  $\lambda_s$ , the average of  $X_s$  is:

$$E(X_s) = \frac{1}{\lambda_s} \quad (20)$$

Accordingly the constraints (CS4), given in (5), with  $\lambda_s$  and  $\lambda_w$  parameters of the exponential distribution become:

$$CS4 : \alpha_t \leq \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}) + \frac{1}{\lambda_s} \quad \forall t = 1, \dots, T \quad (21)$$

$$CS4 : \alpha_t \leq \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}) + \frac{1}{\lambda_s} + \frac{1}{\lambda_w} \quad \forall t = 1, \dots, T \quad (22)$$

### 5.1.3. Average Failure Amount Based Probabilistic Constraints

By considering  $f(x) = \lambda_s \cdot e^{-\lambda_s \cdot x}$ , the constraint (8), for the use of one type of renewable energy source, becomes:

$$\alpha_t \leq \sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} \cdot P_{k,l}) - \left[ \left( \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - \frac{1}{\lambda_s} + \frac{e^{-\lambda_s \cdot \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})}}{\lambda_s} \right) \right] \quad (23)$$

$$\forall t = 1, \dots, T$$

In the case of using two types of renewable sources, constraint (CS5) becomes:

$$\alpha_t \leq \sum_{k=1}^K \sum_{l=1}^{R_k} (V_{k,l} \cdot P_{k,l}) - \left[ \left( \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - \frac{1}{\lambda_s} + \frac{e^{-\lambda_s \cdot \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})}}{\lambda_s} \right) \right] \quad (24)$$

$$- \left[ \left( \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l}) - \frac{1}{\lambda_w} + \frac{e^{-\lambda_w \cdot \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})}}{\lambda_w} \right) \right] \quad \forall t = 1, \dots, T$$

Thus, according to the assumption that  $X_s, X_w$  random variables follow the exponential distribution, the equivalents of the proposed constraints are obtained. In the subsequent section, as the last step, the equivalences of the proposed objective functions are obtained and the stochastic models are built.

## 5.2. Probabilistic Objective Functions

### 5.2.1. OF.1: *Expected Cost of The Probability of Failure*

The objective function OF.1 is introduced in the constraint (11) and the expression of  $P(\sum_{o \in \{s,w\}} X_o < \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}))$  is studied for the constraint (3). Accordingly, for the use of one type of renewable energy (solar, for instance) the objective function OF.1, (11), becomes:

$$\begin{aligned}
 OF.1 : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t} x_{m,t} + hI_{m,t} + w_{m,t} y_{m,t}) \\
 & + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l} P_{k,l}) \\
 & + \sum_{t=1}^T \gamma \cdot (1 - e^{[-\lambda_s \cdot (\alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}))]})
 \end{aligned} \tag{25}$$

Similarly, the objective function proposed for the “contracted renewable energy” (12) is expanded according to the exponential distribution as follows:

$$\begin{aligned}
 OF.1.a : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t} x_{m,t} + hI_{m,t} + w_{m,t} y_{m,t}) \\
 & + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l} P_{k,l}) \\
 & + \sum_{t=1}^T \gamma \cdot (1 - e^{[-\lambda_s \cdot (\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}))]})
 \end{aligned} \tag{26}$$

Considering two types of renewable sources, the objective function given in (11), with the expression of  $P(X_s + X_w < \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l}))$  obtained in (15), becomes:

$$\begin{aligned}
OF.1 : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) \\
& + \sum_{t=1}^T \gamma \cdot \frac{\lambda_s \cdot (1 - e^{-\lambda_w \cdot (Q_t)}) - \lambda_w (1 - e^{-\lambda_s \cdot (Q_t)})}{\lambda_s - \lambda_w}
\end{aligned} \tag{27}$$

where:

$$Q_t = \alpha_t - \sum_{l=1}^{R_{tr}} (V_{tr,l} \cdot P_{tr,l})$$

The objective function developed for the constraint CS3 becomes:

$$\begin{aligned}
OF.1.a : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) \\
& + \sum_{t=1}^T \gamma \cdot \frac{\lambda_s \cdot (1 - e^{-\lambda_w \cdot (Q_t)}) - \lambda_w (1 - e^{-\lambda_s \cdot (Q_t)})}{\lambda_s - \lambda_w}
\end{aligned} \tag{28}$$

where:

$$Q_t = (\sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) + \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l}))$$

### 5.2.2. OF.2 : *Expected Cost of The Average Failure Amount of Renewable Energy*

In the second type of objective function, the expected cost is calculated based on the amount of renewable energy that is not met. Expected failure amount of the renewable energy source (solar energy for instance) is obtained in the constraint (23). Thus, the expected failure amount based objective functions given in (13) are acquired, for the use of one/two type(s) of renewable energy source, as follows:



$$\begin{aligned}
OF.2 : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) \\
& + \sum_{t=1}^T \gamma \cdot \left[ \left( \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - \frac{1}{\lambda_s} + \frac{e^{-\lambda_s \cdot \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})}}{\lambda_s} \right) \right]
\end{aligned} \tag{29}$$

$$\begin{aligned}
OF.2 : Minz = & \sum_{t=1}^T \sum_{m=1}^N (\psi_{m,t}x_{m,t} + hI_{m,t} + w_{m,t}y_{m,t}) \\
& + \sum_{l=1}^{R_k} \sum_{k=1}^K (Vcost_{k,l}P_{k,l}) \\
& + \sum_{t=1}^T \gamma \cdot \left[ \left( \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l}) - \frac{1}{\lambda_s} + \frac{e^{-\lambda_s \cdot \sum_{l=1}^{R_s} (V_{s,l} \cdot P_{s,l})}}{\lambda_s} \right) \right] \\
& + \sum_{t=1}^T \gamma \cdot \left[ \left( \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l}) - \frac{1}{\lambda_w} + \frac{e^{-\lambda_w \cdot \sum_{l=1}^{R_w} (V_{w,l} \cdot P_{w,l})}}{\lambda_w} \right) \right]
\end{aligned} \tag{30}$$

So far, the equivalences of the proposed probabilistic constraints and the objective functions have been obtained with the exponential distribution consideration. Now, the stochastic models can be built by merging the model presented in [23] with the equivalences of the probabilistic constraints and objective functions. The constructed probabilistic models are presented into two categories based on the type of the objective functions in Table 2 and Table 3.

Table 2: The probabilistic models with the Objective Function (OF.1)

Models.	PS1	PS1a	PS2	PS2a	PS3	PS4	PS5	PS6
Obj. Func.	25	26	27	28	25	27	25	27
Pro. Cons.	14	17	15	18	21	22	23	24

The sum of the contracted capacity is already considered in constraints (23) and (24). Therefore, when these constraints are added to the model in [23], the constraint (19) becomes redundant and is removed.

Table 3: The probabilistic models with the Objective Function (OF.2)

Models	PS7	PS8	PS9	PS10	PS11	PS12
Obj. Func.	29	30	29	30	29	30
Pro. Cons.	14	15	21	22	23	24

Therefore, all the proposed probabilistic constraints and objective functions have been expanded by considering the exponential distribution for the random variables which represent the availability of renewable energy sources. Then, the equivalences of them have been merged with the model of [23]. Finally, probabilistic models (named as PS1 to PS12) are obtained to cope with the uncertainty of the renewable energy sources for the single-item lot sizing problems with capacity selection consideration. In the following section, the proposed Mixed Integer Non-Linear Programming models are tested on small size instances and the results are analyzed in detail.

## 6. Numerical Study

In this section, with the aim of validation of the proposed models, different combinations of the proposed constraints and objective functions are tested on an illustrative example and the results are discussed in detail. Since the proposed models involve non linear components, they are solved by LINGO 18.0 on an Intel Core i7 with 1.6 GHz and 16 GB RAM. **It is noteworthy to indicate that the proposed models have been firstly linearized (using piece-wise linear functions) and solved using CPLEX on the same computer. However, the model could not be solved to optimality in an affordable time for small size instances. This experience led us to use the LINGO solver.**

### 6.1. Data Generation

In the numerical example, three machines ( $N=3$ ) are considered and the planning horizon is split into three periods ( $T=3$ ). The machines' powers are  $\phi_m = [8 \ 9 \ 7]$  (KW) and processing times of the machines  $p_m = [5 \ 6 \ 6]$  (min).

Table 4: Time related data of instance N3.T3

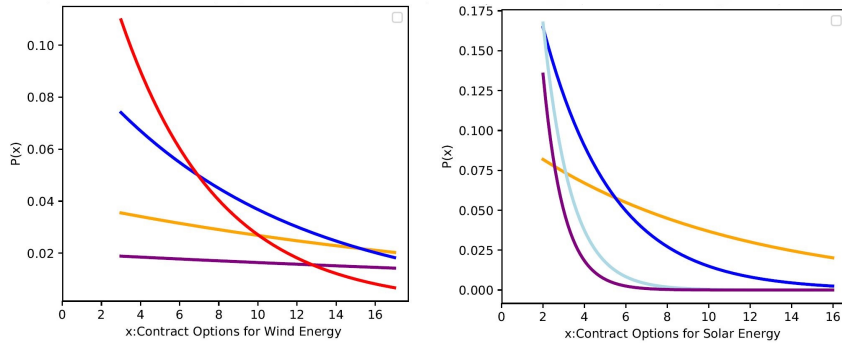
<i>Period (t)</i>	1	2	3
$d_t(\text{piece})$	53	47	75
$L_t(\text{min})$	1080	360	1080
$Co_t(\text{€/KWh})$	0.16	0.08	0.16
$w_{1,t}(\text{€})$	74	52	91
$w_{2,t}(\text{€})$	64	89	99
$w_{3,t}(\text{€})$	57	96	75

Time related data is given in Table 4. In Table 5, contract options for different types of energy sources which are offered by energy supplier and corresponding prices are given. In this example, it is assumed that the price of the traditional energy sources is more expensive than the renewable energy sources. Therefore, it is aimed to constitute a trade-off between the *costly-reliable* and *cheaper - less reliable* options. To conduct a comparative study and to highlight the impact of the probabilistic objective functions and constraints in this study, the same data set presented in [23] is used for the energy, time and machine related data. The penalty cost ( $\gamma$ ) is considered as 10€ for all the models including penalty cost in the objective function.

Table 5: Contract options

Trad.	$Vcost(\text{€})$	Solar	$Vcost(\text{€})$	Wind	$Vcost(\text{€})$
3 KW	75	2 KW	30	3 KW	20
6 KW	82,5	4 KW	35	5 KW	25
9 KW	120	6 KW	40	7 KW	30
12 KW	135	8 KW	50	11 KW	40
15 KW	150	10 KW	60	13 KW	50
18 KW	300	12 KW	85	15 KW	75
24 KW	375	14 KW	120	17 KW	100

As it is pointed out before, the main parameter describing the stochastic nature of the renewable energy sources is  $\lambda_s$  used for solar energy and  $\lambda_w$  used for wind energy. In Figure 3, different availability levels of the wind and solar energy sources are displayed. Accordingly, it can be said that when  $\lambda_w$  values are equal to 0.04 and 0.02, wind energy source is pretty reliable. However, when  $\lambda_w$  is equal to 0.2, the wind energy source is less reliable. The high reliability levels such as  $\lambda_w=0.04$  or 0.02 are introduced to the instance intentionally, in order to analyze the behaviour of the model while selecting the best contract options when the reliability of the renewable source is nearly the same with the deterministic case.



(a) Contract Options for Wind Energy (b) Contract Options for Solar Energy

Figure 3: The curves of the probability-wind/solar power

The reliability of the contract options of solar energy can be interpreted in the same way. While the  $\lambda_s = 0.75$  and  $\lambda_s=1$ , promise less reliability, under the weather conditions where the availability of solar energy is expressed with  $\lambda_s=0.1$ , the solar energy becomes more reliable for the end-users.

Some of the models which can best highlight the impact of the conducted study are tested by taking into account the given instance and the different  $\lambda_w$  and  $\lambda_s$  values which are displayed in Figure 3.

## 6.2. Results of The Model PS1

First of all, the results of the model presented in [23] are recalled in Table 6. In this model, required renewable energy based service level constraint (CS1) is applied to cope with the uncertainty of the renewable energy sources. However, the objective function of the model is as in (10) and the cost of the probable failure in the supply of the renewable energy sources is not considered.

In Table 7, the objective function of the model presented in [23] is replaced with the proposed objective function in (11), thus, the probabilistic cost is added to production and energy purchasing costs. It is evidently seen that when the objective function is extended by adding the non-linear components, the computation time increases. While the model of [23] can be solved in less than 1 second, the computation times of PS1 change between 10 and 20 seconds. When it comes to the contract options, the optimum energy mix which can satisfy the demand under the given service level does not change for the two models. However, it is necessary to indicate that when the penalty cost ( $\gamma$ ) is increased from 10€ to 200€, it is seen that the optimum contract options significantly change. For example, when the service level is 0.5 and  $\lambda_w=0.02$  and the penalty cost is increased from 10€ to 200€, the model PS1 finds 15 KW traditional and 3 KW wind energy mix as the optimum solution instead of the 6-11 mix obtained under the assumption that the penalty cost is 10€ (Table 7). Therefore, the penalty cost has vital importance on the obtained results. Based on the current assumption and results, PS1 gives an insight to the industrial customers about the costs that must be burdened due to the uncertain nature of the renewable energy sources. In Figure 4, the production configuration obtained by the model PS1 is illustrated. As it can be seen, the maximum power amount (17 KW) is demanded in the first period. When the customer prefers to mix 18KW traditional energy and 3 KW energy, this choice keeps the customer 100% safe side since 18 KW, “*risk free*”, traditional energy source always satisfies the energy need of the system. This choice does not include any risk costs, that’s why, the cost of this choice is always 1355.59 € for two models. However, in case of mixing the 6 KW traditional and 11 KW wind

energy sources, under the service level 0.5 and  $\lambda_w=0.04$ , the customer knows that he/she should pay 7.98 € (1166.07-1158.09) more to purchase the risk in the supply. When the reliability is increased from 0.04 to 0.02, this additional cost decreases as expected (1166.07>1162.46) (Table 7). Thus, PS1 can provide better guidance to the industrial customers to generate the optimum energy mix by evaluating the risk cost of different choices.

Table 6: The results of the model in [23]

Service Level	$\lambda_w$	Contract (Tr-W)	Objective Value	CPU(s)
SL=0.5	0.2	15-3	1205.59	<1
	0.1	12-5	1195.59	< 1
	0.04	6-11	1158.09	< 1
	0.02	6-11	1158.09	< 1
SL=0.9	0.2	18-3	1355.59	< 1
	0.1	18-3	1355.59	< 1
	0.04	15-3	1205.59	<1
	0.02	12-5	1195.31	< 1
SL=0.99	0.2	18-3	1355.59	< 1
	0.1	18-3	1355.59	< 1
	0.04	18-3	1355.59	< 1
	0.02	18-3	1355.59	< 1

In Table 7, the other noticeable result is that as long as the service level is increased, the manufacturer tends to mix the renewable energy source with more traditional energy source to be on the safe side. This tendency can be clearly observed in the results of the model where the service level is assumed to be 0.99. Since the values  $\lambda_w=0.04$  or  $\lambda_w=0.02$  guarantee more reliability, under these conditions, the customer can choose more renewable energy sources compared to the less reliable circumstances where the  $\lambda_w=0.2$  or  $\lambda_w=0.1$ . While the customer can guarantee to realize the production with 50% (SL=0.5), by mixing 15 KW traditional and 3 KW wind sources under  $\lambda_w=0.2$ , he can sat-

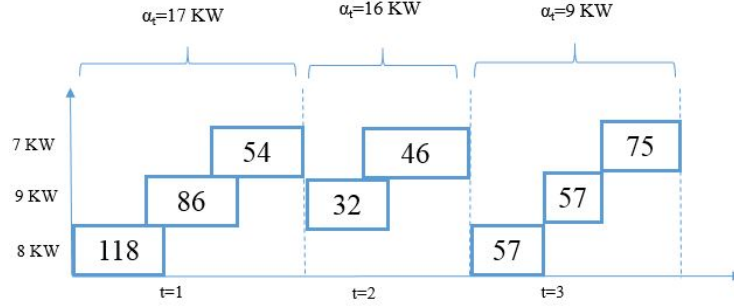


Figure 4: The production configuration of PS1

Table 7: The results of the model PS1

Service Level	$\lambda_w$	Contract (Tr-W)	Objective Value	CPU(s)
SL=0.5	0.2	15-3	1210.7	11
	0.1	12-5	1202.8	17
	0.04	6-11	1166.07	12
	0.02	6-11	1162.46	14
SL=0.9	0.2	18-3	1355.59	12
	0.1	18-3	1355.59	19
	0.04	15-3	1206.75	16
	0.02	12-5	1197.31	11
SL=0.99	0.2	18-3	1355.59	12
	0.1	18-3	1355.59	12
	0.04	18-3	1355.59	11
	0.02	18-3	1355.59	13

isfy the same service level by mixing 6 KW traditional and 11 KW wind source when the  $\lambda_w$  is changed to 0.04. The results for the other service levels can be interpreted in the same way.

Table 8: The results of the model *PS1a*

Service Level	$\lambda_w$	Contract Values (Tr-W)	Objective Value	CPU(s)
SL=0.5	0.2	15-3	1219.12	<1
	0.1	12-5	1207.39	<1
	0.04	6-11	1168.77	<1
	0.02	6-11	1164.01	<1
SL=0.9	0.2	N.F		
	0.1	N.F		
	0.04	N.F		
	0.02	12-5	1198.44	5
SL=0.99	0.2	N.F		
	0.1	N.F		
	0.04	N.F		
	0.02	N.F		

### 6.3. Results of The Model *PS1a*

The results shown in Table 8 allow us to compare two types of service level based chance constraints: PS1 and PS1a. As it is mentioned before, the models PS1a and PS2a help industrial customers to make decisions that can keep them in a safer side compare to PS1 and PS2. The rigidness of the model PS1a is proved by the results displayed in Table 8. First noticeable result is that when the service level is guaranteed by the manufacturers as 0.5, they negotiate for the same mixes which are obtained by model PS1 in Table 7, however, they accept to pay more. In other words, the customer purchases the rigidness by paying more. While the production is feasible under service level 0.9 and 0.99 when the model PS1 is applied (Table 7), when it comes to the results of the model PS1a (Table 8), it is not possible to realize the production by guaranteeing the service level 0.99. The manufacturer can satisfy the demand with the confidence of 90% by mixing the 12 KW traditional and 5 KW wind energy sources when the reliability of wind energy source is 0.02 and the less reliable



conditions ( $\lambda_w=0.2$ ,  $\lambda_w=0.1$ ) make realizing the production impossible.

#### 6.4. Results of The Model PS3

Table 9 shows the results of PS3. The service level is left out of the context and the uncertainty in the availability of the renewable energy source is handled by considering the expected availability of the renewable energy source. The results show that the expected availability based constraint is softer than the service level based chance constraint whose results are exhibited in Table 7. This variant proposes to generate energy mixes composing of *less traditional-more renewable energy sources* as it can be seen in Table 7 and Table 9 when the reliability is defined with  $\lambda_w=0.2$ .

Table 9: The results of the model PS3

$\lambda_w$	Contract (Tr-W)	Objective Value	CPU(s)
0.2	12-5	1207.41	15
0.1	12-5	1202.82	32
0.04	6-11	1166.07	11
0.02	6-11	1162.46	25

#### 6.5. Results of The Model PS5

The model PS5 tested for the same data set. If the customer prefers to negotiate with the energy supplier according to expected failure amount of the renewable energy options, he can reach an agreement with the energy supplier by purchasing more renewable energy source compared to the other types of constraints as shown Table 10. Based on this evidence, it can be interpreted that expected failure amount based constraint can help the industrial customers to generate the best energy mix to sustain their production activities more environment-friendly way.

Table 10: The results of the model PS5

$\lambda_w$	Contract Values (Tr-W)	Objective Value	CPU(s)
0.2	15-3	1210.7	29
0.1	12-7	1207.82	10
0.04	9-11	1200.77	22
0.02	6-13	1172.46	32

Though the nature of the constraints CS4 and CS5 is the same, since the two constraints are developed based on the average amount of renewable energy availability, two approaches have been studied to reflect the stochasticity of the renewable energy. In the first one (based on the average power amount), the expected value of the contract is used to satisfy the energy need. Therefore, the expected power of one or two renewable energy sources are considered as the energy supplied. In the second one (based on the average failure amount), the contracted power of the renewable energy is considered and the stochasticity is taken into account as a failure. Thus, their impact on generating best energy mix under the same weather conditions and objective function is different.

#### 6.6. Results of The Model P7

The results given in Table 7 and Table 11 allow us to compare the impact of the two different types of objective functions since the rest of the models are the same (PS1 and PS7). When the service level is 0.5, the manufacturer has the tendency to generate the energy mix with the smaller amount of wind energy sources compared to the results given in Table 7. The main idea behind this behaviour of the model is that; under a given reliability level, an increase in the amount of the contracted wind energy source causes an increase in the expected failure amount, too. Since the same constraint (14) is applied in two models, both of the combinations (6-11, 12-5) shown in Table 7 and Table 11 under  $\lambda_w=0.04$  satisfy the related constraint. However, increasing amount in the renewable energy portion (11>5) causes an increase in the expected failure

amount, so, the expected cost increases as well. As a result, the model (PS7) combines the traditional energy with less amount of renewable energy compared to (PS1).

Table 11: The results of the model PS7

Service Level	$\lambda_w$	Contract (Tr-W)	Objective Value	CPU(s)
SL=0.5	0.2	15-3	1227.91	6
	0.1	15-3	1217.83	6
	0.04	12-5	1209.63	11
	0.02	6-11	1191.86	6
SL=0.9	0.2	18-3	1377.91	11
	0.1	18-3	1367.83	9
	0.04	15-3	1210.78	12
	0.02	12-5	1202.84	5
SL=0.99	0.2	18-3	1377.91	9
	0.1	18-3	1367.83	7
	0.04	18-3	1360.78	4
	0.02	18-3	1358.23	5

### 6.7. Complexity Analysis on Larger-Size Instances

So far, the results obtained from the models PS1, PS1a, PS3, PS5 and PS7 have been compared based on economical, environmental and reliability aspects. Now, to draw attention to the complexity of the proposed models, a few more numerical experiments on larger-size instances can be conducted.

The models PS1, PS3 and PS5 are tested on two groups of slightly larger instances, which are composed of 4 machines and 4 periods (N4.T4) and 5 machines and 5 periods (N5.T5). For each group, three instances are tested for each model. The computation time is limited to 10 minutes in our tests. If the model reaches a solution within the defined time limit, the computation time is

taken as it is and given in the following table, otherwise, the model is stopped at the end of the time limit and the found solution is used as the reference objective value for further analysis. When the service level based model (PS1) is tested, based on the illustrative study presented in Table 7, the service level is considered as 0.5 and  $\lambda_s=0.02$ , thus, it is intended to see the behavior of the models under relatively less rigid case compared to the case in which service level is considered as 0.9 or 0.99.

The results are presented in Table 12. It had been proved that when the small size instance (N3.T3) is tested, it is seen that the LINGO solver can reach the optimality within short computational times. However, the results shown in Table 12 prove that the models PS1, PS3 and PS5 can not reach the optimum solution for the (N4.T4) and (N5.T5) instances within defined time limit. It is possible to have a feasible solution for some of the instances which include four machines (N4.T4) at the end of 10 minutes, when the instances including 5 machines (N5.T5) are solved, the difficulty increases and any of the instances can not reach any feasible solution after 10 minutes.

Table 12: The results of the probabilistic models for larger instances

Instance	PS1		PS3		PS5	
	Obj.Value	CPU	Obj.Value	CPU	Obj.Value	CPU
4-4_1	1611.17	600	1611.17	600	1220.93.	600
4-4_2	No Sol.	600	No Sol.	600	No.Sol.	600
4-4_3	No Sol.	600	No Sol.	600	No Sol.	600
5-5_1	No Sol.	600	No Sol.	600	No Sol.	600
5-5_2	No Sol.	600	No Sol.	600	No Sol.	600
5-5_3	No Sol.	600	No Sol.	600	No Sol.	600

In Table 13, the instances (N4.T4) are tested by introducing fewer contract options into the model to reduce the complexity. This time, instead of seven options as it is displayed in Table 5, three options for each energy source are presented to the customers. The results show that; reducing the power options

allows to reach a feasible solution for the models PS1 and PS3 for the second and third instance, which could not be solved to feasibility when all the options are included. However, the models still can not reach an optimum solution after 10 minutes even the number of options is reduced.

Table 13: The results of PS1, PS3, PS5 with less contract options

Instance	PS1		PS3		PS5	
	Obj.Value	CPU	Obj.Value	CPU	Obj.Value	CPU
4-4_1	1610.16	600	1610.16	600	1365.99	600
4-4_2	2143.49	600	2143.49	600	No Sol.	600
4-4_3	1514.63	600	1514.63	600	No Sol.	600

#### 6.8. Results of The Models PS2 and PS4

The model PS2 is tested on the data set given in Table 4 and Table 5 and the results are given in Table 14. Since two types of renewable sources are mixed with the traditional energy source, increasing complexity of the developed model causes longer computation times. In most cases, the constructed model can not reach the optimum solution at the end of the time limit and the reference solutions are displayed.

When the service level is increased to 0.9 and the reliability of the renewable energy sources quite low such  $\lambda_s=0.3$  and  $\lambda_w=0.2$ , the production configuration shown in Figure 4 changes slightly (see, Figure 5). When the service level is risen to 0.99, 24 KW is chosen for the traditional energy source, the configuration changes as in Figure 6, thus, the customer can conduct the production activities in a safer way.

When the service level consideration is excluded and the model PS4 is tested on the same instance, as it is seen in Table 15, it is not possible to obtain an optimum solution for some conditions within the defined time limit on the contrary to the results obtained within seconds in Table 9 where single type renewable energy source is mixed with traditional energy source.

Table 14: The results of the model PS2

Service Level	$\lambda_s - \lambda_w$	Contract (Tr-S-W)	Obj. Value	CPU(s)
SL=0.5	0.1- 0.04	6-6-5	1186.01	600
	0.1- 0.02	6-6-5	1184.65	193
	0.3- 0.2	12-2-3	1226.56	600
	0.3- 0.1	9-6-3	1222.46	600
	0.3- 0.02	6-4-7	1185.98	177
	0.75- 0.02	6-6-5	1186.78	106
SL=0.9	0.1- 0.04	9-6-3	1217.19	600
	0.1- 0.02	6-6-5	1184.65	147
	0.3- 0.2	15-2-3	1241.681	600
	0.3- 0.1	15-2-3	1240.75	600
	0.3- 0.02	9-4-5	1217.29	231
	0.75- 0.02	12-2-3	1226.64	88
SL=0.99	0.1- 0.04	15-2-3	1239.85	600
	0.1- 0.02	15-2-3	1239.78	600
	0.3- 0.2	24-2-3	1881.54	600
	0.3- 0.1	24-2-3	1881.54	600
	0.3- 0.02	15-2-3	1239.92	600
	0.75- 0.02	18-2-3	1437.62	600

Therefore, in this section, some of the proposed probabilistic models have been tested on small size instances and the obtained results have been discussed shortly to give better insight to the customers who intend to use the developed models. The most noteworthy result of this effort in this section is that, even though, the models are tested on small-size instances, it is seen that in most cases the model can not reach the optimum solution after 10 minutes. When the models that are particularly thought for the mix of traditional and two types of renewable energy sources, can not reach the optimum solution even for the instance including three machines within the defined time limit (Table 14,

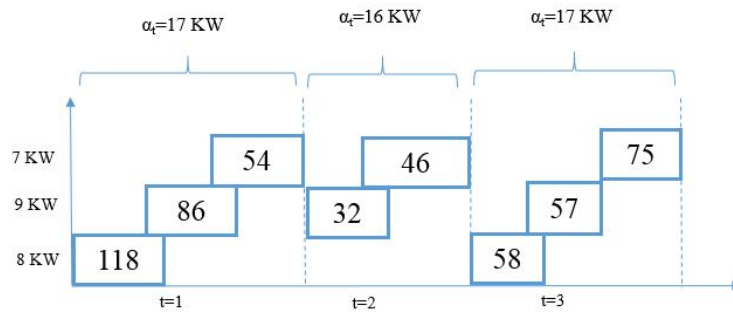


Figure 5: The production configuration of PS2,  $\lambda_s = 0.3, \lambda_w = 0.2, SL = 0.9$

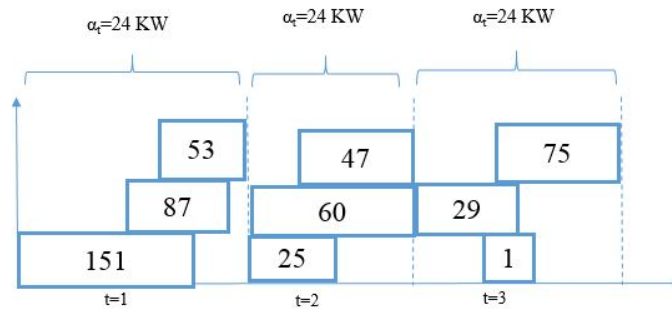


Figure 6: The production configuration of PS2,  $\lambda_s = 0.3, \lambda_w = 0.2, SL = 0.99$

Table 15: The results of the model PS4

$\lambda_s - \lambda_w$	Contract Values (Tr-S-W)	Objective Value	CPU(s)
0.1-0.04	6-6-5	1186.01	600
0.1-0.02	6-6-5	1184.65	176
0.3-0.2	9-4-5	1226.39	600
0.3-0.1	6-6-5	1193.93	196
0.3-0.02	6-6-5	1185.98	84
0.75-0.02	6-6-5	1186.78	64

Table 15). The same complexity is observed in the models which consider the mix of traditional and one type of renewable energy source even the number of

the power options are reduced (see Table 13).

### 7. Case Study: A three-machine flow-shop in a textile company

In this section, to test the proposed probabilistic models, a real workshop, composed of a three-machine flow shop system, of a traditional French textile company is considered. The planning horizon is divided into three periods ( $T=3$ ). The machines' powers are  $\phi_m = [12 \ 8 \ 15]$  (KW) and processing times of the machines  $p_m = [3 \ 6 \ 6]$  (min). In the company, the production, firstly, is planned in weekly manner. Later on, the detailed production plan is established by allocating the weekly demand to days and hours. The demanded production quantities for these three periods are  $d_t = [45 \ 55 \ 60]$ . The electricity consumption is based on TOU strategy. Accordingly, the electricity prices (€) considered are (0.17, 0.13) for the corresponding ON-OFF periods inspired by a tariff offered by a retailer in a French electricity market. Given the environment of the study, in which prices of energy sources do not correspond to current prices, the price of traditional power is directly taken from market prices and the price of renewable sources is artificially generated (see Table 5). The availability of renewable energy source is assumed  $\lambda_w = 0.02$ .

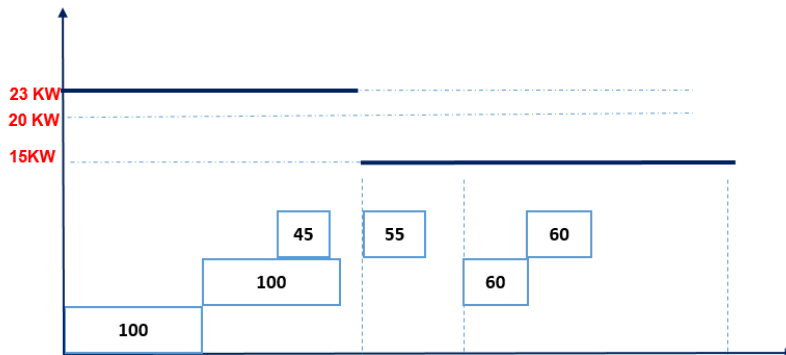


Figure 7: The production configuration of PS7,  $SL = 0.99$



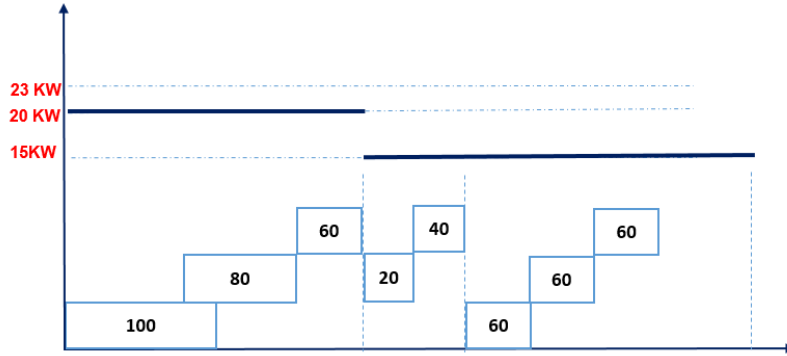


Figure 8: The production configuration of PS11

The handled case is tested with the three probabilistic models: PS7, PS9 and PS11 and the results are displayed in Table 16. The production configurations obtained presented in Figure 7 and Figure 8. The squares serve as machines and the numbers inside them represent the found optimum production quantities for each machine and period. The power demand for each period is displayed at the top of each interval with the solid lines. By displaying these configurations, it is aimed to observe the impact of different probabilistic constraints on the production plan. PS7 gives an insight to the customers who desire to build the production plan under renewable energy uncertainty by guaranteeing a certain service level ( $=0.99$ , for the related case). This model yields that the energy mix composed of 24 KW traditional and 3 KW wind energy source is the optimum energy mix. When the customer prefers to take into account the expected failure amount in the supply of wind energy, as PS11, the model is resulted with the mix of 15 KW traditional and 7 KW wind energy source.

The manufacturers can approach the model PS7 when they need to be on the safer side and to guarantee their customers that the production will be realized with the 99% service level. If they are not subject to this type of strict condition, they can apply PS11 and satisfy the demand under more environmental friendly conditions by mixing the traditional energy source with more renewable energy

Table 16: The results of the model PS7, PS9 and PS11

			N3.T3_C7	N3.T5_C7	N5.T5_C7	N5.T5_C3
Service Level	$\lambda_s$	CPU(s)	CPU(s)	CPU(s)	CPU(s)	CPU(s)
PS7	0,5	0.2	14	216	600	600
		0.1	19	167	600	600
		0.04	27	367	600	600
		0.02	20	372	600	600
	0,9	0.2	9	109	600	600
		0.1	10	148	600	600
		0.04	17	207	600	600
		0.02	14	176	600	600
	0,99	0.2	12	116	600	600
		0.1	15	121	600	600
		0.04	10	120	600	600
		0.02	11	127	600	600
PS9	0.2	27	321	600	600	
	0.1	24	229	600	600	
	0.04	24	460	600	600	
	0.02	19	398	600	600	
PS11	0.2	18	177	600	600	
	0.1	15	91	600	600	
	0.04	55	247	600	600	
	0.02	26	213	600	600	

source.

In Table 16, under the column named with N3.T3\_C7, the computation time of the considered real case, including three machines, is presented. The models PS7, PS9 and PS11 can be solved less than 1 minute. In order to give a better sense about the complexity of the models, they are tested for the same working environment by augmenting the number of the periods to 5. As it can be

observed under the column named with N3\_T5\_C7, although the computation time increases compare to the instance N3\_T3\_C7 the models can be solved to optimality before ten minutes. When two more machines are added to the workshop and tested for the same models (N5\_T5\_C7), in this case, any solution cannot be obtained within the defined time limit. The results below the column (N5\_T5\_C3), (C3, indicates that the number of contracts is three) clearly show that this complexity of the models continues even if the number of contracts has been reduced.

### 7.1. Managerial Insights and Limitations

The proposed models in this work allow the managers:

- to reach an agreement for the exact electricity power they need to realize the production plan by evaluating the unavailability risks in the supply of the renewable sources. Contrary to the work of [24], since a relationship is established between the production plan and the contract values, it prevents the purchase of unnecessary amounts of energy even under renewable energy related uncertainty.
- to use energy sources more efficiently. Contrary to the work proposed in [22], which assumes that renewable energy sources are supplied uninterruptedly, and to the work proposed in [24], which only considers the unit energy cost, when the current work is tested for the same instance, it yields a higher cost since it takes into account several aspects that are not considered in [22] and [24] (stochastic supply of renewable energy and electricity power limit). Therefore, our work yields a cost that adheres much better to real costs faced by the managers. Moreover, the production and energy mix configurations correspond best to the real life problem.
- to communicate better with the clients. The uncertainty of renewable energy sources can be translated to a service level and this approach can help managers if the production plan can be realized for a given service level and a weather condition. If the production is feasible under given

conditions, the service level can be used to communicate with the clients, ie. the demand will be satisfied by 90% under given weather conditions, by considering green production targets.

- to evaluate economical, environmental and reliability aspects of different energy mixes.
- to meet governmental objectives for green production.
- to measure and minimize the risks related to the utilisation of the renewable energy sources. The managers can plan their production activities based on the estimated risks in the supply of renewable energy sources.

In their current form, the proposed models in this work can be used by enterprises which have the intention to conduct their production activities in a more energy efficient and environmental friendly way. As a perspective, efficient (meta) heuristic approaches can be developed and the proposed models can become useful for large sized of production facilities.

## 8. Conclusion

In this study, three groups of probabilistic constraints and two types of objective functions are developed to deal with the uncertain nature of the renewable energy sources during the power purchasing procedure. Firstly, the developed constraints and objective functions are explained in detail and their general forms are given. Then, their equivalences are obtained based on the exponential distribution.

The proposed constraints and the objective functions allow to obtain several mathematical models for single-item capacitated lot sizing problem with capacity selection aspect. The proposed probabilistic models help the decision makers to cope with the intermittent nature of the renewable energy sources while deciding the power contract and the size of the production lots simultaneously. The provided decision-making tools encourage the industrial customers

to conduct their production activities in a more environmental friendly way and contribute to establish better communication with their customers.

It is necessary to indicate that there are still some rooms to improve in this work. Firstly, the probabilistic mathematical models are proposed based on the exponential distribution consideration. Different probability distributions such as Weibull, Normal, Rayleigh, etc. can be tailored into the probabilistic constraints and objective functions proposed in this paper **by following the steps presented in Section 3 and Section 4.**

The obtained Mixed Integer Non-Linear Programming models give solutions for small-size instances. For large size instances, to deal with the complexity of the models, it would be interesting to propose appropriate heuristic approaches or approximation algorithms that can cope with the complexity of the handled NP-hard problem. Therefore the proposed probabilistic models would become more applicable for the real life production environments composed of more machines and organised based on longer planning horizons.

**In this study, the on-site energy generation, which would be significantly beneficial in reducing energy costs and in following a more environmental friendly production policy for the industrial customers, is not considered. Integrating on-site generation into the problem requires taking into account many features such as the charge and discharge rates of energy storage systems, the cost of storing energy, the dynamics between the purchasing and selling prices of renewable energy sources. Therefore, the on-site generation is an interesting extension of this work to tackle the case of companies having on-site generation and storage systems. It is also possible to take into account interruptions in the supply of the traditional energy which would be a fruitful subject to manage the production in the cities/countries where such electricity interruptions are frequent.**

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## References

- [1] EIA, International energy outlook 2019 (2019).
- [2] P. Griffin, R. Heede, The carbon majors database: Cdp carbon majors report 2017, CDP Report.
- [3] M. Mohammadi, S. Fatemi Ghomi, B. Karimi, S. Torabi, Mip-based heuristics for lotsizing in capacitated pure flow shop with sequence-dependent setups, *International Journal of Production Research* 48 (10) (2010) 2957–2973.
- [4] M. Babaei, M. Mohammadi, S. FatemiGhomi., Lot sizing and scheduling in flow shop with sequence-dependent setups and backlogging, *International Journal of Computer Applications* 29 (8) (2011) 52–59.
- [5] R. Ramezani, M. Saidi-Mehrabad, P. Fattahi, MIP formulation and heuristics for multi-stage capacitated lot-sizing and scheduling problem with availability constraints, *Journal of Manufacturing Systems* 32 (2) (2013) 392–401.
- [6] N. Mortezaei, N. Zulkifli, Integration of lot sizing and flow shop scheduling with lot streaming, *Journal of Applied Mathematics* 2013.
- [7] D. Gong, Y. Han, J. Sun, A novel hybrid multi-objective artificial bee colony algorithm for blocking lot-streaming flow shop scheduling problems, *Knowledge-Based Systems* 148 (2018) 115–130.
- [8] Y.-Y. Han, D. Gong, X. Sun, A discrete artificial bee colony algorithm incorporating differential evolution for the flow-shop scheduling problem with blocking, *Engineering Optimization* 47 (7) (2015) 927–946.

- [9] Y. Han, D. Gong, Y. Jin, Q.-k. Pan, Evolutionary multi-objective blocking lot-streaming flow shop scheduling with interval processing time, *Applied Soft Computing* 42 (2016) 229–245.
- [10] Y. Han, D. Gong, Y. Jin, Q. Pan, Evolutionary multiobjective blocking lot-streaming flow shop scheduling with machine breakdowns, *IEEE transactions on cybernetics* 49 (1) (2017) 184–197.
- [11] R. Ramezani, M. Saidi-Mehrabad, Hybrid simulated annealing and mip-based heuristics for stochastic lot-sizing and scheduling problem in capacitated multi-stage production system, *Applied Mathematical Modelling* 37 (7) (2013) 5134–5147.
- [12] M. Mahdih, Integrated capacitated lot sizing and scheduling problems in a flexible flow line, Ph.D. thesis, University of the West of England (2013).
- [13] H. M. Villadiego, J. E. C. Arroyo, A. G. dos Santos, An iterated greedy heuristic for simultaneous lot-sizing and scheduling problem in production flow shop environments, in: *European Conference on Evolutionary Computation in Combinatorial Optimization*, Springer, 2014, pp. 61–72.
- [14] S. Torkaman, S. F. Ghomi, B. Karimi, Multi-stage multi-product multi-period production planning with sequence-dependent setups in closed-loop supply chain, *Computers & Industrial Engineering* 113 (2017) 602–613.
- [15] N. Absi, S. Dauzère-Pérès, S. Kedad-Sidhoum, B. Penz, C. Rapine, Lot sizing with carbon emission constraints, *European Journal of Operational Research* 227 (1) (2013) 55–61.
- [16] Y. Yu, Z. Hong, C. Chu, L. Liang, A dynamic lot sizing model with carbon emission constraint and multi-mode production: A general property and a polynomially solvable case, in: *Networking, Sensing and Control (ICNSC)*, 2013 10th IEEE International Conference on, IEEE, 2013, pp. 413–417.
- [17] A. Akbalik, C. Rapine, Single-item lot sizing problem with carbon emission under the cap-and-trade policy, in: *2014 International Conference on*

- Control, Decision and Information Technologies (CoDIT), IEEE, 2014, pp. 030–035.
- [18] M. J. R. Helmrigh, R. Jans, W. van den Heuvel, A. P. Wagelmans, The economic lot-sizing problem with an emission capacity constraint, *European Journal of Operational Research* 241 (1) (2015) 50–62.
- [19] T. Zouadi, A. Yalaoui, M. Reghioui, Hybrid manufacturing/remufacturing lot-sizing and supplier selection with returns, under carbon emission constraint, *International Journal of Production Research* 56 (3) (2018) 1233–1248.
- [20] O. Masmoudi, A. Yalaoui, Y. Ouazene, H. Chehade, Lot-sizing in flow-shop with energy consideration for sustainable manufacturing systems, *IFAC-PapersOnLine* 28 (3) (2015) 727–732.
- [21] M. Rodoplu, T. Arbaoui, A. Yalaoui, Energy contract optimization for the single item lot sizing problem in a flow-shop configuration and multiple energy sources, *IFAC-PapersOnLine* 51 (11) (2018) 1089–1094.
- [22] M. Rodoplu, T. Arbaoui, A. Yalaoui, A fix-and-relax heuristic for the single-item lot-sizing problem with a flow-shop system and energy constraints, *International Journal of Production Research* 58 (21) (2020) 6532–6552.
- [23] M. Rodoplu, T. Arbaoui, A. Yalaoui, Single item lot sizing problem under renewable energy uncertainty, *IFAC-PapersOnLine* 52 (13) (2019) 18–23.
- [24] O. Masmoudi, A. Yalaoui, Y. Ouazene, H. Chehade, Lot-sizing in a multi-stage flow line production system with energy consideration, *International Journal of Production Research* 55 (6) (2017) 1640–1663.
- [25] O. Masmoudi, A. Yalaoui, Y. Ouazene, H. Chehade, Solving a capacitated flow-shop problem with minimizing total energy costs, *The International Journal of Advanced Manufacturing Technology* 90 (9-12) (2017) 2655–2667.



- [26] A. Charnes, W. W. Cooper, Chance-constrained programming, *Management science* 6 (1) (1959) 73–79.
- [27] C. Lin, Stochastic single-source capacitated facility location model with service level requirements, *International Journal of Production Economics* 117 (2) (2009) 439–451.
- [28] L. Li, Z. B. Zabinsky, Incorporating uncertainty into a supplier selection problem, *International Journal of Production Economics* 134 (2) (2011) 344–356.
- [29] M. Najafi, A. Ahmadi, H. Zolfagharinia, Blood inventory management in hospitals: Considering supply and demand uncertainty and blood transshipment possibility, *Operations Research for Health Care* 15 (2017) 43–56.
- [30] K. Liu, Z.-H. Zhang, Capacitated disassembly scheduling under stochastic yield and demand, *European Journal of Operational Research* 269 (1) (2018) 244–257.
- [31] Ö. B. Kınay, B. Y. Kara, F. Saldanha-da Gama, I. Correia, Modeling the shelter site location problem using chance constraints: A case study for istanbul, *European Journal of Operational Research* 270 (1) (2018) 132–145.
- [32] M. Noorizadegan, B. Chen, Vehicle routing with probabilistic capacity constraints, *European Journal of Operational Research* 270 (2) (2018) 544–555.
- [33] C. Gicquel, J. Cheng, A joint chance-constrained programming approach for the single-item capacitated lot-sizing problem with stochastic demand, *Annals of Operations Research* 264 (1-2) (2018) 123–155.
- [34] A. Kamjoo, A. Maheri, G. A. Putrus, Chance constrained programming using non-gaussian joint distribution function in design of standalone hybrid renewable energy systems, *Energy* 66 (2014) 677–688.

- [35] J. D. Vergara-Dietrich, M. M. Morato, P. R. Mendes, A. A. Cani, J. E. Normey-Rico, C. Bordons, Advanced chance-constrained predictive control for the efficient energy management of renewable power systems, *Journal of Process Control*.
- [36] P. Beraldi, A. Violi, G. Carrozzino, M. E. Bruni, The optimal electric energy procurement problem under reliability constraints, *Energy Procedia* 136 (2017) 283–289.
- [37] T. Mühlpfordt, T. Faulwasser, V. Hagenmeyer, A generalized framework for chance-constrained optimal power flow, arXiv preprint arXiv:1803.08299.
- [38] W. Van Ackooij, Chance constrained programming: with applications in energy management, Ph.D. thesis, Ecole Centrale Paris (2013).
- [39] F. S. Gazijahani, S. N. Ravadanegh, J. Salehi, Stochastic multi-objective model for optimal energy exchange optimization of networked microgrids with presence of renewable generation under risk-based strategies, *ISA transactions* 73 (2018) 100–111.
- [40] N. Nikmehr, S. N. Ravadanegh, Reliability evaluation of multi-microgrids considering optimal operation of small scale energy zones under load-generation uncertainties, *International Journal of Electrical Power & Energy Systems* 78 (2016) 80–87.
- [41] T. Tran, A. Smith, Stochastic optimization for integration of renewable energy technologies in district energy systems for cost-effective use, *Energies* 12 (2019) 533.
- [42] Y. Kun, K. Zhang, Y. Zheng, L. Dawei, W. Ying, Y. Zhenglin, Irregular distribution of wind power prediction, *Journal of Modern Power Systems and Clean Energy* 6 (6) (2018) 1172–1180.