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Double-type player games and proportional cross-evaluation allocation method for collaborative logistics

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Abstract In collaborative logistics, multiple shippers may collaborate with each other by consolidating their transportation requests and negotiate a contract with a single carrier to serve the requests at a favorable tariff. In this collaboration, agents of one type (the shippers) cannot increase their payoffs without the participation of an agent of the other type (the carrier). One key issue for such collaboration is the cost allocation among the shippers. Motivated by this real need, we introduce a new class of games called double-type player games and propose a cost/payoff allocation method for them. Each double-type player game has two types of players or agents: service requestors and service providers. We prove that the core of any game in this class is not empty and apply the concept of cross-evaluation originating from data envelopment analysis to generate a fair cost/payoff allocation among the players in such a game. More particularly, we propose a new and simple method called proportional cross-evaluation method for generating an allocation in the semi-core with some important properties. It is based on self-evaluation and peer-evaluations of the contribution of each player to the grand coalition. As applications, this method is applied to the cost allocation of double-type player games appearing in shipper collaboration in both truckload transportation and less-than-truckload transportation. It is compared with several popularly used allocation methods by numerical experiments. The results show that our method outperforms those methods in guaranteeing an allocation in the core.

Keywords: collaborative logistics; cooperative games; double-type player games; cross-evaluation.

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1. Introduction

In recent years, collaboration among shippers or carriers has been playing an increasingly important role in transportation industry (Yilmaz and Savaseneril, 2012; Li et al., 2015; Aloui et al., 2021). To achieve the economies of scale, more and more shippers or carriers have formed collaborative alliances/coalitions by consolidating their transportation requests or sharing their transportation resources in order to reduce costs, improve the responsiveness to market demands, and capture new business opportunities. One key issue is how to share the total cost or profit generated by a coalition among its participants (Guajardo and Rönnqvist, 2016; Cruijssen et al. 2007; Cuervo et al., 2016).

Motivated by this real need in collaborative logistics, we introduce a new class of games called double-type player games and propose a cost/payoff allocation method for them. In collaborative logistics, two types of agents exist. One is service requestors and the other is service providers. Figure 1 illustrates the two types of agents. Agents of one type cannot increase their value without the participation of an agent of the other type. One example of such collaboration is shipper collaboration, where multiple shippers collaborate with a carrier in providing and serving transportation requests. In such collaboration, the shippers are transportation service requestors while the carrier acts as a service provider. The shippers consolidate/bundle their transportation requests and negotiate a contract with the carrier to serve the requests at a favorable price/tariff. Certainly, without the participation of the carrier, the transportation requests of each shipper cannot be served and consequently its payoff will not increase. This type of collaboration is also referred to as shipper-carrier collaboration hereafter since one carrier is also involved in it.

To model such collaboration, we introduce double-type player games in which multiple players of one type cooperate with a single player of the other type. This class of games are closely related to ours is that of big boss games introduced by Muto et al. (1988). Compared with the big boss games, the games studied in this paper are more general. For example, to model a single-sided combinatorial auction with an auctioneer (seller) and multiple bidders (buyers) by a big boss game, the bidders are required to be “substitute” so that the marginal product (value) of any subset of bidders exceeds the sum of their individual marginal products (values) (de Vries et al., 2007). However, all single-sided combinatorial auctions can be modelled by our double-type player games, because more bidders participating in the auction will not decrease its total payoff (the auctioneer’s payoff plus the payoff of all bidders), implying that the game derived from the auction is super-additive. Since any big boss

game is super-additive, it is also a double-type player game, but a double-type player game may not be a big boss game. In fact, the big boss games form a proper subclass of our double-type player games. Besides, when each request can be represented by a lane with an origin and a destination in a transportation network in truckload transportation, our game can be considered an extension of the lane covering game (LCG) (Hezarkhani et al., 2014; Özener and Ergun, 2008). LCG is only applicable to truckload transportation, whereas our game can be applied to both full truckload transportation and less than truckload transportation. Moreover, LCG only considers collaboration among shippers, whereas our game considers both collaboration among shippers and collaboration between the shippers and the carrier. Our game is reduced to a lane covering game if we remove the carrier and restrict all transportation requests to full truckload requests.

In this paper, we also explore properties of the double-type player games and prove that the core of any double-type player game is nonempty. Moreover, profit (or cost) allocation among players in such a game is investigated. Previously, an allocation method based on self-evaluation and peer-evaluations of the contribution of each player to the grand coalition in a game was proposed for generating an allocation in the core of the game (Chen, 2016). However, that method requires all coalitional values and is quite time-consuming when the number of players is large and the problem for calculating each coalitional value is NP-hard. For this reason, we propose a new and simple method called proportional cross-evaluation method for generating an allocation in the semi-core of a game by only using limited but essential information about the characteristic function of the game. In self-evaluation, each player maximizes its own contribution to the grand coalition subject to semi-core constraints. In peer-evaluation, each player evaluates the contributions of the other players without deteriorating its own contribution. The payoff of each player is then allocated according to its average contribution obtained by its self-evaluation and the peer evaluations of all other players.

Although our allocation method proposed in this paper is based on the same concept of cross-evaluation of Chen (2016), they are different in three aspects. Firstly, we replace the core by the semi-core in the cross-evaluation of the contribution of each player to address the problem of the impossibility of obtaining all coalitional values and the computational complexity. Secondly, a much simpler proportional rule instead of the center of gravity is used to determine the peer-evaluation contribution of each player. Thirdly, our cross-evaluation value can be calculated by an analytical expression, which is much simpler than that of Chen (2016). To distinguish with that of Chen (2016),

the value (allocation method) proposed in this paper is called proportional cross-evaluation value hereafter.

Compared with the previous allocation methods, the proportional cross-evaluation value has the following four advantages. Firstly, it considers self- and peer-evaluation contributions of each player to the grand coalition and is thus more easily accepted by the players involved. Secondly, all the players use the same rule for both self-evaluation and peer-evaluations, and the allocation obtained by our method is thus fair to all the players. Thirdly, it is always in semi-core. Fourthly, it has some important properties, such as efficiency, symmetry, null player and weakly coalitional monotonicity. Note that this new allocation method can be applied not only to the double-type player games introduced in this paper but also to any super-additive games with nonempty semi-core. In the following, when defining the proportional cross-evaluation value for a game, we assume it is super-additive and has a nonempty semi-core.

The proposed cost/profit allocation method can be applied to collaborative logistics in both full truckload transportation and less-than truckload transportation. By modeling the shipper-carrier collaboration as a double-type player game, we prove that the game is super-additive and its core is always nonempty. In the context of collaboration in less-than truckload transportation, since the vehicle routing problem for calculating the coalitional cost (value) of each coalition of this game is NP-hard and its number of possible coalitions is exponential, generating an allocation by any core-based method is time-consuming and even impossible in practice, especially when the number of players is large. In this specific application context, our proposed proportional cross-evaluation allocation method has demonstrated its advantages for the allocation of the cost savings gained by the collaboration among all shippers and the carrier. Numerical experiments show that our method has the best performance compared with some existing methods, such as alternate cost avoided method, proportional to stand-alone costs method, and the Shapley value, in guaranteeing an allocation in the core.

The rest of this paper is organized as follows. Section 2 introduces the new class of cooperative games with two types of players. In Section 3, the proportional cross-evaluation allocation method is proposed for such games with certain important properties of the method presented. In Section 4, the new allocation method is applied to double-type player games appearing in shipper-carrier

collaboration. Section 5 reports and analyzes the results of computational experiments on the collaborative logistics games. Section 6 concludes this paper with remarks for future research.

2. Cooperative games with two types of players in collaborative logistics

In this paper, we investigate cooperative games in collaborative logistics with two types of players, service providers and service requestors. The first type, noted by type I hereafter, has only one player, whereas the second type, noted by type II, has multiple players. The first type is either of service player or of service requestor. If the first type is of service player, the second type is of service requestor, and vice versa. Figure 1(a) and (b) illustrates two possible cases of such games. The collaboration described by such games commonly exists in collaborative logistics. For example, in collaboration between multiple shippers and a carrier, multiple shippers consolidate their transportation requests and procure transport services from the carrier. In such collaboration, if we consider transportation requests of shippers as service requests, then shippers are service requestors, whereas the carrier is the service provider.

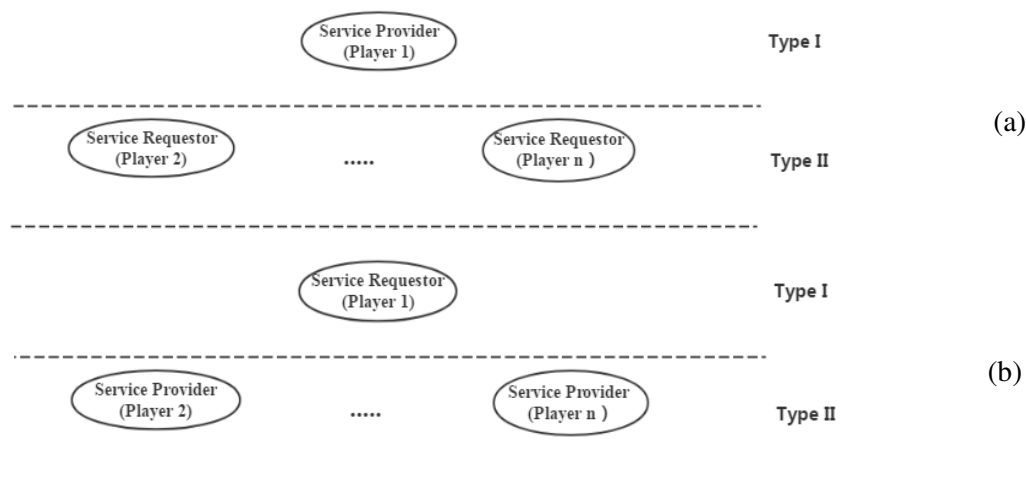


Fig. 1. Cooperation structure with two types of players.

In Figure 1(a), the single service provider of Type I is denoted by player 1, and the service requestors of type II are denoted by players 2 to n . Each player of Type II has service requests, which need to be served by the player of Type I who may also have its own requests. That is, the player of Type I as a service provider must not only serve the requests of the players of type II but also its own requests. There is no more value added for the collaboration among the players of type II (the service requestors) because without the player of type I their requests cannot be served together. In such collaboration, the shippers may submit their transportation requests to a web-based platform who

procure transportation services from a carrier to serve these requests. Cooperation between the service requestors and the service provider may achieve economies of scale and consequently reduce their total transportation costs. A similar explanation can be given to the cooperation structure in Figure 1(b), where the single service requestor of Type I is denoted by player 1, such as a single shipper who has multiple requests, and the service providers of type II are denoted by players 2 to n , such as multiple carriers who fulfill the requests from the single shipper.

To model such cooperation structure, we introduce a new class of cooperative games called double-type player games which have two types of players. Firstly, a cooperative game is defined by the pair (N, v) , where $N = \{1, \dots, n\}$ is the set of players and v is a characteristic function, which is a real-valued function defined on all subsets of N with $v(\emptyset) = 0$. v is also called payoff function of the game. Each subset S of N , i.e. $S \subseteq N$, is called a *coalition* of the game. Particularly, $N = \{1, \dots, n\}$ which contains all players is called the grand coalition. The value of each coalition S is defined by $v(S)$. Denote the set of all players by $N = \{1, 2, \dots, n\}$, where player 1 represents the single player of Type I and players 2 to n represent the players of Type II. For each player i , denote its stand-alone payoff by $v(\{i\})$, which is the payoff that the player can obtain without the cooperation of any other players. Each game in this class satisfies the following conditions.

(i) The cooperation among any players of Type II without the participation of the player of Type I does not create any additional value. That is, the payoff of any coalition of players of Type II is the sum of their stand-alone payoffs, that is,

$$v(S) = \sum_{i \in S} v(\{i\}), \quad \forall S \subseteq N \setminus \{1\}. \quad (1)$$

(ii) The game is super-additive, that is,

$$\text{for any } S, T \subseteq N, S \cap T = \emptyset \text{ implies } v(S) + v(T) \leq v(S \cup T). \quad (2)$$

Super-additive condition is very commonly in the cooperative game because it can make the players have the incentives to cooperate with each other to increase their value. It is a very weak condition that can be satisfied. In our context, condition (ii) implies that the cooperation among any players of Type II with the participation of the player of Type I may create an additional value so that the players of Type II have incentives to cooperate with each other via the player of Type I.

The above two conditions (properties) define a class of double-type player games that commonly exist in real life. One example of such games can be found in collaborative logistics mentioned above,

where shippers may collaborate with a single carrier to serve their transportation requests. Another example is a combinatorial auction where an auctioneer (seller) sells multiple goods or services to one or multiple buyers (bidders). If either the bidders or the auctioneer is absent, the auction cannot happen. Moreover, if the auction achieves social efficiency (i.e., globally optimizes the total payoff of the auctioneer and bidders), then more bidders participate in the auction, a higher (or at least the same) total payoff can be gained by the coalition of auctioneer and bidders because more intensive competition among bidders will improve resource allocation efficiency. Consequently, the underlying double-type player game of this auction is super-additive.

For this game, an allocation among all players is a real-valued vector $x = (x_1, \dots, x_n)$, where x_i is the payoff allocated to player i , $i \in N$. An allocation x is efficient if $\sum_{i \in N} x_i = v(N)$ and individually rational if $x_i \geq v(\{i\})$ for all $i \in N$. It is called an imputation if it is both efficient and individually rational. The core of the game (N, v) is defined as $C(N, v) = \{x \in R^n \mid \sum_{i \in N} x_i = v(N), \sum_{i \in S} x_i \geq v(S), \forall S \subset N\}$, which contains all imputations under which no coalition has a value greater than the total payoff of its members. For this new class of double-type player games, firstly we have the following important property:

Theorem 1. *The core of each game in this new class of double-type player games is nonempty.*

Proof. See Appendix.

Theorem 1 shows that the core always exists for any double-type player game in which type I has only one player and type II has multiple players. This implies this game has a stable payoff allocation according to the definition of the core.

3. Proportional cross-evaluation method for payoff allocation of double-type player games

One important issue for a cooperative game is how to allocate its total payoff (value) of the grand coalition among its players in a fair way. If the core of the game is not empty, it is expected that the allocation is in the core. A payoff allocation method is called core-based if it generates one or multiple allocations in the core if the latter is not empty. Well known core-based allocation methods include Nucleolus (Schmeidler, 1969), the core center (González-Díaz and Sánchez-Rodríguez, 2007), and the Alexia value (Tijs et al., 2011).

Although core-based allocation methods are attractive, the calculation of an allocation in the core of a game by applying such a method is time-consuming when the number of players is large, because it requires all coalitional values (the characteristic function values of all coalitions), but the number of coalitions is exponential with respect to the number of players. Even though our assumption (i) helps a little bit in reducing the number of computations, the final number is still exponential (namely order 2^{n-1} instead of 2^n). Moreover, for some complex problems such as the vehicle routing game given in Göthe-Lundgren et al. (1996), the calculation of all coalitional costs (transportation costs of all coalitions of customers) is very time-consuming because the underlying capacitated vehicle routing problem is NP-hard. Moreover, in some cases, only part of these coalitional values or costs can be obtained.

For the two reasons, we propose a new allocation method by applying the concept of cross-evaluation originating from data envelopment analysis (Sexton et al., 1986; Doyle and Green, 1994) for the payoff/cost allocation of our double-type player games. The basic idea is that the total payoff of a game is allocated among its players based on the cross-evaluation of the contribution of each player to the grand coalition, where the contribution of each player is evaluated by itself (self-evaluation) and all other players (peer evaluations). Since both self- and peer-evaluations of the contribution of each player to the grand coalition are considered, the allocation is fair to all players in some sense. This concept was applied to payoff allocation of cooperative games in our previous work (Chen, 2016). However, our previously proposed allocation method has two weaknesses. The first one is that the method requires finding the center of gravity of a polytope but this is time-consuming and usually NP-hard (Elbassioni and Tiwary, 2012). The second one is that the method requires knowing all coalitional values, which may be impossible for some real cooperative games. Even if all these values can be obtained, when the number of players n is large, the computation of (2^n-1) coalitional values may be very time-consuming (Xu et al., 2009; Nguyen and Thomas, 2016). For example, when the number of players is 20, the number of coalitional values will reach 1,048,575. If the problem for obtaining each coalition's value is a mixed integer programming (MIP) problem as in vehicle routing games or other collaborative logistics games, we need to solve 1,048,575 MIP problems, this is intractable. As an alternative, we propose a new and simple method to generate an allocation in the semi-core for our double-type player games.

3.1 Proportional cross-evaluation value

Instead of finding an allocation in the core, this new method tries to find an allocation in the semi-core of a game, because the semi-core is much easier to obtain in practice as it requires only the values of $2n+1$ most important coalitions, which retain the most essential information about the characteristic function of the game (Flisberg et al., 2015; Guajardo and Rönnqvist, 2016). Note that the allocation generated by well-known ACA method for a cost game is also in the semi-core when it is nonempty and the game is sub-additive (Young, 1994). For any double-type player game, we will prove the allocation obtained by our method is always in the semi-core. Moreover, numerical experiments in Section 5 will demonstrate that in most cases the allocation obtained by our method is also in the core.

By considering only non-blocking constraints for coalitions $\{i\}$ and $N\setminus\{i\}$ for all $i \in N$, the semi-core is formally defined as follows.

Definition 1. The semi-core of a game (N, v) is defined as

$$SC(N, v) = \{x \in R^n \mid \sum_{i \in N} x_i = v(N), x_i \geq v(\{i\}), \sum_{i \in N \setminus \{j\}} x_i \geq v(N \setminus \{j\}), \forall i, j \in N\}. \quad (3)$$

In formula (3), $\sum_{i \in N} x_i = v(N)$ represents the efficiency constraint, and $x_i \geq v(\{i\})$ for any represent the individual rationality constraints. $\sum_{i \in N \setminus \{j\}} x_i \geq v(N \setminus \{j\})$ for any represent the group rationality constraints for all coalitions containing $|N| - 1$ players where $|N|$ is the number of players. All these constraints are called semi-core constraints hereafter. Obviously, $C(N, v) \subseteq SC(N, v)$, i.e., $SC(N, v)$ contains core.

Before introducing the new payoff allocation method, we define the self-evaluation, peer-evaluation contributions and proportional cross-evaluation value of a player to the grand coalition.

Definition 2. The self-evaluation contribution of a player i , $E_{ii}(v)$, is the maximum possible payoff that the player can obtain without violating the semi-core constraints.

Definition 3. The peer-evaluation contribution of player j with respect to player i , denoted by $E_{ij}(v)$, is the contribution of player j to the grand coalition evaluated from the point of view of player i with neither deteriorating its self-evaluation contribution and nor violating the semi-core constraints.

Definition 4. The proportional cross-evaluation value of each player, i.e., player i , $\bar{E}_i(v)$ in a game is the average of its self-evaluation contribution $E_{ii}(v)$ and the peer-evaluation contributions of this player with respect to all other players $E_{ji}(v)$, $j \neq i$, that is, $\bar{E}_i(v) = \frac{\sum_{j=1}^n E_{ji}(v)}{n}$.

By definition 2, the self-evaluation contribution of each player i , denoted by E_{ii} , can be obtained by solving the following linear program in which player i pursues its maximum possible payoff under the condition that (x_1, \dots, x_n) is in the semi-core.

$$\begin{aligned}
& E_{ii} = \max x_i \\
\text{s. t.} \quad & \sum_{j \in N} x_j = v(N) \\
& x_k \geq v(\{k\}), \forall k \in N \\
& \sum_{j \in N \setminus \{k\}} x_j \geq v(N \setminus \{k\}), \forall k \in N
\end{aligned} \tag{4}$$

According to model (4), we have the following theorem about the self-evaluation contribution.

Theorem 2. The self-evaluation contribution of each player i , $E_{ii}(v)$, is $v(N) - v(N \setminus \{i\})$ if the semi-core of the game is nonempty.

Proof. According to the definition of the semi-core, evidently, $v(\{i\}) \leq x_i \leq v(N) - v(N \setminus \{i\})$ for any $x \in SC(v)$. Therefore, if we can find an allocation (x_1, \dots, x_n) with the payoff $v(N) - v(N \setminus \{i\})$ for player i that satisfies all the constraints of model (4), then we prove this theorem. If $x_i = v(N) - v(N \setminus \{i\})$, then $x \in SC(v)$ can be written as $\sum_{j \in N \setminus \{i\}} x_j = v(N \setminus \{i\})$, $v(k) \leq x_k$ and $\sum_{j \in N \setminus \{k\}} x_j \geq v(N \setminus \{k\})$ for any $k \in N$ and $k \neq i$. The last set of constraints can be rewritten as $\sum_{j \in N \setminus \{k\}} x_j = \sum_{j \in N} x_j - x_k = v(N) - x_k \geq v(N \setminus \{k\})$, i.e., $x_k \leq v(N) - v(N \setminus \{k\})$ for any $k \in N$ and $k \neq i$. Therefore, $x \in SC(v)$ can be written as $\sum_{j \in N \setminus \{i\}} x_j = v(N \setminus \{i\})$ and $v(k) \leq x_k \leq v(N) - v(N \setminus \{k\})$. Such x exists if $v(N \setminus \{i\}) \leq \sum_{k \in N \setminus \{i\}} (v(N) - v(N \setminus \{k\}))$. In the following, we prove that this condition is satisfied. Let $n = |N|$. For any $x \in SC(v)$, we have $\sum_{k \in N} \sum_{j \in N \setminus \{k\}} x_j \geq \sum_{k \in N} v(N \setminus \{k\})$. Given that $\sum_{k \in N} \sum_{j \in N \setminus \{k\}} x_j = (n-1) \sum_{k \in N} x_k = (n-1)v(N)$, $(n-1)v(N) \geq \sum_{k \in N} v(N \setminus \{k\})$, i.e., $v(N \setminus \{i\}) \leq \sum_{k \in N \setminus \{i\}} (v(N) - v(N \setminus \{k\}))$ for all $i \in N$. If $SC(N, v)$ is not empty, then the condition is valid. Therefore, the self-evaluation contribution of player i is $v(N) - v(N \setminus \{i\})$. \square

Since the core of any double-type player game is nonempty, its semi-core is also nonempty, the following proposition is evident according to Theorem 2.

Proposition 1. *The self-evaluation contribution of each player i in the double-type player game is given by $v(N) - v(N \setminus \{i\})$.*

After obtaining the self-evaluation contribution, each player i also needs to evaluate the contribution of any other player j ($j \neq i$) to the grand coalition. According to Theorem 2, the self-evaluation contribution $E_{ii}(v)$ of each player i is given by its maximal possible payoff $v(N) - v(N \setminus \{i\})$, i.e., the marginal contribution of the player to the grand coalition. When player i evaluates the contribution of any other player j , it will take its self-evaluation contribution $v(N) - v(N \setminus \{i\})$ as its payoff, and allocate the remaining payoff $v(N \setminus \{i\})$ among all other players. After allocating $z_i(v) = v(N) - v(N \setminus \{i\})$ to player i and excluding player i from the allocation of the remaining payoff, we derive the reduced semi-core $SC^{-i}(v) = \{x^{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) \mid \sum_{j \in N \setminus \{i\}} x_j = v(N) - z_i(v), v(k) \leq x_k, \sum_{j \in N \setminus \{i, k\}} x_j \geq v(N \setminus \{i, k\}) \text{ for any } k \in N \text{ and } k \neq i\} = \{x^{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) \mid \sum_{j \in N \setminus \{i\}} x_j = v(N \setminus \{i\}), v(k) \leq x_k \leq v(N \setminus \{k\}), \text{ for any } k \in N \text{ and } k \neq i\}$. So the peer-evaluation contributions $E_{ij}(v), j \neq i$ must be defined as a point in $SC^{-i}(v)$. We define the peer-evaluation contributions by applying a proportional rule as follows.

Let $x_j = v(\{j\}) + \rho_i \times (v(N) - v(N \setminus \{j\}) - v(\{j\}))$, for any $j \neq i$, where $\rho_i \in [0, 1]$ is a proportion depending on i , such that $\sum_{j \in N \setminus i} x_j = \sum_{j \in N \setminus i} v(\{j\}) + \rho_i \times \sum_{j \in N \setminus i} (v(N) - v(N \setminus \{j\}) - v(\{j\})) = v(N \setminus i)$, i.e., $\rho_i = \frac{v(N \setminus \{i\}) - \sum_{j \in N \setminus \{i\}} v(\{j\})}{\sum_{j \in N \setminus \{i\}} (v(N) - v(N \setminus \{j\}) - v(\{j\}))}$.

This proportion ρ_i has the following properties:

1) If $\rho_i = 0$, $x_j = v(\{j\})$, i.e., each player $j \neq i$ gets its minimal value (stand-alone value) in $SC^{-i}(v)$.

2) If $\rho_i = 1$, $x_j = v(N) - v(N \setminus \{j\})$, i.e., each player $j \neq i$ gets its maximal value in $SC^{-i}(v)$.

Let $r_j(v) = v(N) - v(N \setminus \{j\}) - v(\{j\})$, the difference between the marginal contribution and stand-alone value of player j . For any super-additive game, $r_j(v) \geq 0$ for any $j \in N$. With the proportional rule, the peer-evaluation contribution of player $j \neq i$ with respect to player i , $E_{ij}(v)$, is defined as follows:

$$\begin{aligned} E_{ij}(v) &= v(\{j\}) + \rho_i \times r_j(v) \\ &= v(\{j\}) + \frac{v(N \setminus \{i\}) - \sum_{j \in N \setminus \{i\}} v(\{j\})}{\sum_{j \in N \setminus \{i\}} (v(N) - v(N \setminus \{j\}) - v(\{j\}))} \times (v(N) - v(N \setminus \{j\}) - v(\{j\})) \quad (5) \end{aligned}$$

for any player i and any $j \neq i, j \in N$.

Evidently, by the above definition, the sum of the peer-evaluation contributions of all other players $j, j \in N \setminus \{i\}$ with respect to player i is $\sum_{j \in N \setminus \{i\}} E_{ij}(v) = v(N \setminus \{i\})$, which is exactly the remaining value of the grand coalition after subtracting player i 's self-evaluation value.

For any double-type player game, since players cannot collaborate without the player of Type 1 (i.e., player 1), we have the following corollary about peer-evaluation contribution.

Corollary 1. *For the double-type player game, $E_{1j}(v) = v(\{j\})$ for any $j \neq 1$.*

After defining the self- and peer-evaluation contributions of all players, the proportional cross-evaluation value of each player i ($i \in N$) can be defined by the following equation (6).

$$\begin{aligned} \bar{E}_i(v) &= \frac{\sum_{j=1}^n E_{ji}(v)}{n} \\ &= \frac{v(N) - v(N \setminus \{i\}) + \sum_{j=1, j \neq i}^n (v(\{i\}) + \rho_j \times (v(N) - v(N \setminus \{i\}) - v(i)))}{n} \end{aligned} \quad (6)$$

where $\rho_j = \frac{v(N \setminus \{j\}) - \sum_{k \in N \setminus \{j\}} v(\{k\})}{\sum_{k \in N \setminus \{j\}} (v(N) - v(N \setminus \{k\}) - v(\{k\}))}$ if $\sum_{k \in N \setminus \{j\}} (v(N) - v(N \setminus \{k\}) - v(\{k\})) \neq 0$, $\rho_j = 0$ otherwise, for all $j = 1, \dots, n$.

The following theorem shows that the allocation defined by the proportional cross-evaluation value is in the semi-core under mild conditions.

Theorem 3. *For any super-additive game with nonempty semi-core, its proportional cross-evaluation value is in the semi-core.*

Proof. According to Theorem 2, for any super-additive game with nonempty semi-core, we have $E_{ii}(v) = v(N) - v(N \setminus \{i\})$.

Moreover, the peer-evaluation contribution value of player j with respect to player i , $E_{ij}(v)$, is given by the formula $E_{ij}(v) = v(\{j\}) + \frac{v(N \setminus \{i\}) - \sum_{j \in N \setminus \{i\}} v(\{j\})}{\sum_{j \in N \setminus \{i\}} (v(N) - v(N \setminus \{j\}) - v(\{j\}))} \times (v(N) - v(N \setminus \{j\}) - v(\{j\}))$ for any player i and any $j \neq i, j \in N$. Obviously, $E_i(v) = (E_{i1}(v), \dots, E_{ii}(v), \dots, E_{in}(v)) \in SC(v)$.

Since the proportional cross-evaluation value of each player i , $\bar{E}_i(v)$, is defined as its average cross contribution, i.e., $\bar{E}_i(v) = \frac{\sum_{j=1}^n E_{ji}(v)}{n}$, $\bar{E}(v) = (\bar{E}_i(v), i \in N)$ is also in the semi-core if the latter is not empty because the semi-core $SC(v)$ is convex and $E_i(v) \in SC(v)$ for all $i \in N$.

□

According to Theorems 1 and 3, the following corollary is obvious.

Corollary 2. *The proportional cross-evaluation value of the double-type player game is in its semi-core.*

3.2 Properties of the proportional cross-evaluation value

In the following, we prove that the proportional cross-evaluation value has some important properties, such as efficiency, symmetry, null player, and weakly coalitional monotonicity.

Proposition 2. (Efficiency) *The proportional cross-evaluation value is efficient.*

Proof. According to the definitions of self-evaluation, peer-evaluation and proportional cross-evaluation, i.e., Definitions 2, 3 and 4, we have $\sum_{i \in N} E_{ji} = v(N)$. Since $\bar{E}_i(v) = \frac{\sum_{j=1}^n E_{ji}(v)}{n}$,

$$\sum_{i \in N} \bar{E}_i = \frac{\sum_{i \in N} \sum_{j \in N} E_{ji}}{n} = \frac{\sum_{j \in N} \sum_{i \in N} E_{ji}}{n} = \frac{\sum_{j \in N} v(N)}{n} = v(N). \quad \square$$

Proposition 3. (Symmetry) *For the proportional cross-evaluation value, if player i and j are equivalent in the sense that $v(S \cup \{i\}) = v(S \cup \{j\})$ for any $\forall S \subseteq N \setminus \{i, j\}$, we have $\bar{E}_i = \bar{E}_j$.*

Proof. If the condition of this proposition holds, from Theorem 2, we have $E_{ii} = E_{jj}$; and from Definition 3 and formula (5), we have $E_{ik} = E_{jk}$ and $E_{ki} = E_{kj}$ for all $k \in N \setminus \{i, j\}$. Thus, according to Definition 4 and formula (6), we have $\bar{E}_i = \bar{E}_j$. \square

Proposition 4. (Null player) *For the proportional cross-evaluation value, if j is a null player with $v(S \cup \{j\}) = v(S)$ for any $S \subseteq N \setminus \{j\}$, we have $\bar{E}_j = 0$.*

Proof. From Proposition 2, we have $\sum_{i \in N} \bar{E}_i = v(N)$. Moreover, since the proportional cross-evaluation value is in the semi-core, it satisfies $\sum_{i \in N \setminus \{j\}} \bar{E}_i \geq v(N \setminus \{j\}) = v(N)$. Then, we have $\bar{E}_j = 0$.

\square **Proposition 5. (Weakly coalitional monotonicity).** *The proportional cross-evaluation value \bar{E} satisfies weakly coalitional monotonicity.*

Proof. See Appendix.

4. Cost allocation in collaborative logistics

4.1. Shipper-carrier collaborative game

Collaborative logistics has recently emerged as an effective approach to consolidate logistic flows

beyond logistics platforms. This approach can reduce the repositions of empty vehicles and increase the fill rates of vehicles. Most collaborative logistics studied in the literature belong to horizontal collaboration among multiple actors of the same type in logistics operations, such as collaboration among shippers and collaboration among carriers. In this section, we investigate collaborative logistics with two types of players: (1) service requestors (or request providers) and (2) service providers. Shipper–carrier collaboration (SCC) is a typical example of such collaborative logistics, where shippers provide their transportation requests to a carrier who serves these requests. A collaboration among shippers without the participation of the carrier cannot add any values because the requests are not served. However, with the participation of the carrier, the collaboration among the shippers and carrier can realize the cost reduction where shippers procure transportation services from the carrier to serve their requests.

Different from the previous studies, we model SCC by a double-type player game. For multiple shippers and a carrier in the collaboration, we consider the carrier as the player of Type I and shippers as players of Type II. Each shipper has a set of transportation requests to offer to the carrier who may also has its own transportation requests to serve. In this game, multiple shippers of Type II cannot add their values through collaboration without the participation of the carrier of Type I. Each player i has its stand-alone cost, denoted by $c(i)$. For the carrier, $c(i)$ is the cost of serving its own requests without serving any request got from the shippers in the collaboration. For each shipper, $c(i)$ is the cost of serving its own requests individually by a carrier it has chosen without any collaboration with other shippers in the collaboration. The shippers of Type II can cooperate with the carrier of Type I to reduce their cost by consolidating their requests and offering the consolidated requests to the carrier. Such collaboration exists in both truckload transportation and less-than truckload transportation.

This shipper-carrier collaboration (SCC) game is a cost game defined by a pair (N, c) , where $N = \{1, \dots, n\}$ is the set of players composed of one carrier and $n-1$ shippers and c is the characteristic cost function. In this game, the carrier is denoted by player 1, and $n-1$ shippers are denoted by player 2 to player n , respectively. For any coalition $S \subseteq N$, if S contains player 1 (the carrier), $c(S)$ can be obtained by solving a vehicle routing problem as described hereafter. Otherwise, if S does not contain player 1, $c(S) = \sum_{i \in S} c(\{i\})$, where $c(\{i\})$ is the stand-alone cost of shipper i . For any $1 \in S \subseteq N$ and $i \in N \setminus S$, it is assumed that $c(S) + c(\{i\}) \geq c(S \cup \{i\})$, i.e., it is beneficial for shipper i and carrier-shipper coalition S to form a bigger coalition. This assumption is mild since the grand

carrier-shipper coalition N will not be formed if it does not hold.

4.2. Properties of the SCC cost game

As we know, the cost game (N, c) is sub-additive if $c(S) + c(T) \geq c(S \cup T)$ for all $S, T \subset N$, $S \cap T = \emptyset$. *Subadditivity* indicates that if two disjoint coalitions of players form a single larger coalition they will have a lower or equal total cost compared with the situation where they act individually. In this case of subadditivity, all players have an incentive to cooperate. Analogously, we have the following proposition.

Proposition 6. *The SCC cost game is sub-additive.*

Proof. See Appendix. □

Definition 6. A cost game (N, c) is monotone if its characteristic function c is monotone, i.e., $c(S) \leq c(T)$ for $S \subsetneq T \subseteq N$.

Proposition 7. *The SCC cost game may be not monotone.*

Proof. See Appendix. □

Definition 7. A cost game (N, c) is convex if its characteristic function c is concave (or submodular), i.e., $c(S \cup \{k\}) - c(S) \geq c(T \cup \{k\}) - c(T)$ for all $k \in N$ and $S \subseteq T \subseteq N \setminus \{k\}$ or $c(S \cup T) + c(S \cap T) \leq c(S) + c(T)$ for $S, T \subseteq N$.

Proposition 8. *The SCC cost game (N, c) may be not convex.*

Proof. See Appendix. □

Because this study focuses on payoff (profit) allocation, we convert the SCC cost game into a *cost-saving game* with the characteristics function:

$$v(S) = \sum_{i \in S} c(\{i\}) - c(S) \text{ for all } S \subseteq N. \quad (7)$$

where $v(S)$ can be interpreted as the cost saving value of coalition S . According to formula (7), the cost-saving value for any single player and any coalition formed by only shippers equals zero. That is, $v(i) = 0$ for any $i \in N$, and $v(S) = 0$ for any $S \subseteq N \setminus \{1\}$. Moreover, according to Proposition 6, we can easily prove that the SCC cost-saving game is super-additive. Therefore, we can have the following corollary obviously.

Corollary 3. *The core of the SCC cost game is nonempty.*

4.3. An example of the SCC cost game

To illustrate this cost game and its cost allocation, we take a simple example of shipper-carrier collaboration with one carrier (Player 1) and two shippers (Players 2 and 3), where Players 2 and 3 cannot collaborate without the participation of Player 1. Each player has three requests corresponding to the delivery of goods from a city distribution center (CDC) to three different locations. Player 1 has requests 1a, 1b, and 1c. Player 2 has requests 2a, 2b, and 2c. Player 3 has requests 3a, 3b, and 3c. The capacity of each vehicle is 200. It is assumed that all deliveries start from the same CDC. Considering each position as a point in the map of the city with two coordinates, the CDC is (54.72, 19.66) where 54.72 is its x -coordinate and 19.66 is its y -coordinate. Table 1 gives the destinations (x, y) and the demands of all requests.

Table 1

Destinations and demands of all requests

	x -Coordinate	y -Coordinate	Demand
Player 1 (Carrier)			
Depot	54.72	19.66	–
Request 1a	13.86	25.11	38.69
Request 1b	14.93	61.60	20.02
Request 1c	25.75	47.33	29.39
Player 2 (Shipper 1)			
Request 2a	84.07	35.17	4.79
Request 2b	25.43	83.08	3.70
Request 2c	81.43	58.53	27.54
Player 3 (Shipper 2)			
Request 3a	24.35	54.97	39.96
Request 3b	92.93	91.72	47.70
Request 3c	35.00	28.58	7.50

For each coalition of this game, its value (cost) can be obtained by solving a capacitated vehicle routing problem. After solving all related vehicle routing problems, we obtain all coalitional costs as $c(1)=81.7182$, $c(2)=117.287$, $c(3)=137.1477$, $c(1,2)=140.4186$, $c(1,3)=158.1278$, $c(2,3)=137.1477$, and $c(1,2,3)=158.1278$. Consequently, by formula (7), we can obtain the corresponding cost-saving

values as $v(1)=0$, $v(2)=0$, $v(3)=0$, $v(1,2)=58.5867$, $v(1,3)=60.7381$, $v(2,3)=0$, and $v(1,2,3)=178.0251$. By applying formula (6), we can obtain the proportional cross-evaluation value of the cost savings game as (83.2312, 46.8518, 47.9421). This value is in the semi-core of the game. Since the semi-core coincides with the core for this 3-player game and its core is nonempty, the proportional cross-evaluation value is also in the core.

For the purpose of comparison, we compare the allocation obtained by our method with those obtained respectively by the alternate profit sought method (APS) (Straffin and Heaney, 1981), the proportional to stand-alone costs method (PSC), and the Shapley value in Table 2. The APS method first allocates to each player j its separable profit (cost-saving value) $m_j = v(N) - v(N \setminus \{j\})$. Since $\sum_{j \in N} m_j$ is usually larger than $v(N)$, i.e., some players are over-allocated with profit m_j , $j \in N$, the over-allocated part of profit m_j must be reimbursed (paid back) by player j to the grand coalition. The PSC considered in this paper is based on the proportional rule (Flisberg et al., 2015) which allocates the costs among all players based on the stand-alone cost of each player, that is, $x_j = c(\{j\}) / \sum_{i \in N} c(\{i\}) \times c(N)$, where $c(\{j\})$ is the stand-alone cost of player j . By replacing $c(N)$ with $v(N)$, we can use it to allocate cost savings in a cost savings game. We call such method proportional to stand-alone costs method (PSC).

Table 2.

Allocation for four methods and the core

	<i>Player 1</i>	<i>Player 2</i>	<i>Player 3</i>
PCE	83.2312	46.8518	47.9421
APS	76.4145	50.3436	51.2670
PSC	43.2776	62.1147	72.6328
Shapley	79.2292	48.8601	49.9358

The core of this game is given by the set $\{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 178.0251, x_1 + x_2 \geq 58.5867, x_1 + x_3 \geq 60.7381, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\}$. For this cost saving game, the allocations obtained by APS, PSC and the Shapley value are also in the core. However, the proportional cross-evaluation value is closer to the Shapley value than APS and PSC, which can also be validated by many other randomly generated examples. In the next section, we will show that our cost

allocation method performs better than the other three methods in certain aspects using computational experiments.

5. Computational experiments

In this section, through numerical experiments on a large number of randomly generated game instances appearing in collaborative logistics, we will evaluate the performance of our proportional cross-evaluation method for profit (or cost saving) allocation compared with several popularly used methods in the literature: alternate cost avoided method, proportional to stand-alone costs method, and the Shapley value (Narayanam and Narahari, 2011). We first investigate a SCC in truckload transportation (TL), where multiple shippers consolidate/bundle their requests and negotiate a lower price with a single carrier to serve the requests. This SCC is related to the lane covering problem and the lane covering game (Hezarkhani et al., 2014). We then study a SCC in less than truckload transportation (LTL) in city logistics where each transportation request corresponds to the delivery of goods from a city distribution center to a customer. This SCC also involves multiple shippers and a single carrier.

5.1. SCC in truckload transportation

In this collaboration, the transportation network has 30 nodes (locations) randomly and uniformly generated on the square region $[0, 100] \times [0, 100]$; each shipper has 60 full truckload transportation requests represented by lanes. Each lane has its original node and destination node randomly generated from the 30 nodes. It is assumed that the carrier does not have its own transportation request. The transportation cost from one node to another is simply set as their Euclid distance. Eight sets of game instances with the number of players varying from 3 to 10 are randomly generated. Each set contains 50 instances. All coalitional values of each game in the instances can be obtained by solving a series of lane covering problems. The results of comparison of our proposed method with other three allocation methods are given in Table 3.

Table 3

Number of games with payoff allocation in the core for SCC in TL

Players	APS	PSC	Shapley	PCE
3	50	50	50	50

4	50	31	49	50
5	50	15	48	50
6	50	8	49	50
7	50	2	48	50
8	50	1	48	50
9	50	0	48	50
10	50	0	44	50

In this table, each row corresponds to a set of 50 games randomly generated with the number of players given in the first column. For each set, column 2 to 5 give the number of games whose payoff allocation determined by APS, PSC, Shapley value and PCE, respectively, is in the core. From the table, we can see our proportional cross-evaluation allocation method and APS method have the best performance in terms of stability interpreted by the core, whereas PSC has the worst performance. As the number of players increases, the number of allocations in the core obtained by PSC method decreases drastically, while that number slightly decreases for the Shapley value. Compared with the Shapley value, our proportional cross-evaluation value is much easier to be calculated especially when the number of players is large. Although our method and APS both achieves the best stability for the SCC in truckload transportation, the former method is more appealing conceptually since it is based on the self and peer-evaluation of the contribution of each player to the grand coalition. In the next subsection, we will show our method outperforms APS for randomly generated game instances of SCC in less truckload transportation.

5.2. SCC in less than truckload transportation

The shipper-carrier collaboration problem in less-than truckload transportation considered here has the same structure as that of the example SCC problem in Section 4.3 except that their data such as the number of shippers and the requests of each shipper are different. We consider two scenarios: (1) each player (shipper or carrier) has one request and (2) each player has two requests. We only consider one or two requests for each player is due to high complexity of capacitated vehicle routing problems to solve in order to obtain all coalitional values of a SCC cost game in less than truckload transportation. The capacity of each vehicle is set to 200, the demand of each request is randomly generated according to the uniform distribution defined on $[0, 50]$, and the delivery destination of each request is randomly generated according to the uniform distribution defined on the square region

$[0, 100] \times [0, 100]$. In the first scenario, 7 sets of instances with the number of players varying from 3 to 9 are randomly generated. Each set contains 50 instances. The computer we used is ThinkCentre with 32G memory and Intel I7-8700 CPU operated under Windows 10 professional version. We called the function “intlinprog” in the software of Matlab R2015b for solving all vehicle routing problems involved. Since the capacitated vehicle routing problem is NP-hard, the computation time for verifying an allocation in the core for an instance game increases very quickly with the number of players. For example, when the number of players is 10, this computation time for all 50 instances is 186,560 seconds. The results of comparison of our method with other three allocation methods under this scenario are given in Table 4.

Table 4

Number of games with payoff allocation in the core for SCC in LTL – scenario 1

Players	APS	PSC	Shapley	PCE
3	50	28	50	50
4	50	26	50	50
5	50	17	50	50
6	49	15	46	50
7	50	13	45	50
8	49	12	41	50
9	45	6	35	48
10	48	5	37	50

From Table 4, we can see our proportional cross-evaluation allocation method has the best performance in terms of stability interpreted by the core, whereas PSC has the worst performance. Compared with the Shapley value, our proportional cross-evaluation value is much easier to be calculated.

Similarly, we randomly generate 6 sets of 50 instances in the second scenario. Because it is very time-consuming for instances with more than 8 players, we only obtain the results until 8 players. Table 5 present the results of comparison of our method with other three allocation methods under this scenario.

Table 5

Number of games with payoff allocation in the core for SCC in LTL – scenario 2

Players	APS	PSC	Shapley	PCE
---------	-----	-----	---------	-----

3	50	43	50	50
4	50	36	47	50
5	50	25	41	50
6	49	20	45	50
7	50	13	47	50
8	50	11	43	50

The results in Table 5, similar to those in Table 4, demonstrate the superiority of our proportional cross-evaluation method compared with the other three methods in terms of stability. The results in both tables show that our method performs well for cost-saving allocation of SCC games in both truckload transportation and less than truckload transportation, because for almost all randomly generated instances the allocation determined by our method is in the core. Besides, after checking the results, we find that the allocations obtained by our method and APS are much closer to Shapley value than that obtained by PSC method. Compared with PSC, our method has an advantage that it considers not only the cost of the grand coalition and the stand-alone cost of each player but also the costs of all coalitions excluding one player. More importantly, our method considers both each player's self-evaluation and all other players' peer-evaluations of its contribution to the grand coalition in determining its payoff to each player.

6. Conclusions

This paper reports our study on a new class of games arising in collaborative logistics with two types of players. In such a game, the collaboration among players of one type cannot create any value without the participation of a player of the other type. Such games commonly exist in auctions, collaborative logistics, and supply chain collaboration. However, previous studies rarely considered this kind of games. We first study this new class of games and prove that the core of any game in this class is nonempty. Since for such games appearing in collaborative logistics, the calculation of all coalitional values, which involves NP-hard vehicle routing problems, is very time consuming and even impossible, we propose a new allocation method, called proportional cross-evaluation allocation method, which is quite simple but can guarantee its allocation in the semi-core.

The proportional cross-evaluation allocation method proposed in this paper considers both self-evaluation and peer-evaluations of the contribution of each player to the grand coalition. The

allocation generated by this method has some important properties, such as efficiency, symmetry, null player, and weakly coalitional monotonicity. It is fair to all players in some sense and can be more easily accepted by them because all the players apply the same rules to their self-evaluation and peer-evaluations. Compared with core-based allocation methods, the computation complexity of the proportional cross-evaluation method is much lower because its allocation can be calculated by a closed-form expression. Although the Shapley value can be calculated by a close-form expression, it is not easy to compute when the number of players is large, because it needs to calculate the marginal contribution of each player for all permutations of players entering the game. Besides, the Shapley value may be not in the core even not in the semi-core. Furthermore, according to our numerical experiments, the proportional cross-evaluation allocation method performs better than the alternate profit sought method, proportional to stand-alone costs method, and the Shapley value, in terms of generating an allocation in the core.

Two directions to extend the present work in future are possible. One is to study the axiomatic characterization of the proportional cross-evaluation value. The other is to apply the proportional cross-evaluation allocation method to more general double-type player games with multiple service requestors and multiple service providers appearing in collaborative logistics and supply chain collaboration.

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Appendix:

Proof of Theorem 1.

Proof. Set the payoff of Player 1 (i.e., the player of type I) to $x_1 = v(N) - \sum_{i \in N \setminus \{1\}} v(\{i\})$, and the payoff of any other player to its stand-alone value, i.e., $x_i = v(\{i\}), \forall i \neq 1$, then we have $\sum_{i \in N} x_i = x_1 + \sum_{i \in N \setminus \{1\}} x_i = v(N)$. This indicates that the efficiency constraint of the core $\sum_{i \in N} x_i = v(N)$ and individual rationality constraints $x_i \geq v(\{i\})$ are satisfied by x . Moreover, for any coalition, two cases may happen: Case 1: $1 \in S$, i.e., S contains player 1. In this case, we have $\sum_{i \in S} x_i = x_1 +$

$\sum_{i \in S \setminus \{1\}} x_i = v(N) - \sum_{i \in N \setminus \{1\}} v(\{i\}) + \sum_{i \in S \setminus \{1\}} v(\{i\}) = v(N) - \sum_{i \in N \setminus S} v(\{i\}) = v(N) - v(N \setminus S)$.
 Since (N, v) is super-additive, we have $v(S) + v(N \setminus S) \leq v(N)$, so $\sum_{i \in S} x_i = v(N) - v(N \setminus S) \geq v(S)$. Case 2: $1 \notin S$, i.e., S does not contain player 1. In this case, the payoff of S is the sum of the stand-alone values of all players in S , which is equal to $\sum_{i \in S} v(\{i\})$, so $\sum_{i \in S} x_i = \sum_{i \in S} v(\{i\}) = v(S)$. Therefore, the coalitional rationality constraints $\sum_{i \in N \setminus \{j\}} x_i \geq v(N \setminus \{j\})$ are valid for all sub-coalitions of N . As a result, the allocation x is in the core, so the core of the game is nonempty. \square

Proof of Proposition 5.

The weakly coalitional monotonicity property is defined as follows: Let φ be an allocation rule and G^n be the set of all n -player games with the super-additivity. Let $v, w \in G^n$, and let $T \subseteq N$ be such that $w(T) > v(T)$ and, for each $S \neq T$, $w(S) = v(S)$. φ satisfies weak coalitional monotonicity if $\sum_{i \in T} \varphi_i(w) \geq \sum_{i \in T} \varphi_i(v)$.

Let φ be the proportional cross-evaluation allocation rule defined on G^n , N be the set of players of v and w , and let $T \subseteq N$ be a coalition such that $w(T) > v(T)$, and for any $S \neq T$, $w(S) = v(S)$.

1) When $T = N$, by Proposition 2, we have $\sum_{i \in T} \bar{E}_i(w) = \sum_{i \in N} \bar{E}_i(w) = w(N) > v(N) = \sum_{i \in T} \bar{E}_i(v)$.

2) When $T \subset N$, two cases may happen.

Case 1: $|N|=2$, that is, both games v and w have only two players

In this case, we can easily obtain $\sum_{i \in T} \bar{E}_i(w) \geq \sum_{i \in T} \bar{E}_i(v)$ according to the formula (6).

Case 2: $|N|>2$, that is, both games v and w have more than two players

$$\text{Let } \rho_i^w = \frac{w(N \setminus \{i\}) - \sum_{k \in N \setminus \{i\}} w(\{k\})}{\sum_{k \in N \setminus \{i\}} (w(N) - w(N \setminus \{k\}) - w(\{k\}))} \text{ and } \rho_i^v = \frac{v(N \setminus \{i\}) - \sum_{k \in N \setminus \{i\}} v(\{k\})}{\sum_{k \in N \setminus \{i\}} (v(N) - v(N \setminus \{k\}) - v(\{k\}))} .$$

Case 2 can be further divided into two subcases.

subcase 2a: $|T|=|N|-1$, that is, T contains $|N|-1$ players. Denote by player l the player not in T , then $T=N \setminus \{l\}$.

If $i \in T$, we have $w(N \setminus \{i\}) = v(N \setminus \{i\})$. The self-evaluation values of player i in the two games have $E_{ii}(w) = w(N) - w(N \setminus \{i\}) = v(N) - v(N \setminus \{i\}) = E_{ii}(v)$ for all $i \in T$.

For any $j \in T$ with $j \neq i$ and any $k \in T$, since $w(N \setminus \{i\}) = v(N \setminus \{i\})$, $w(\{k\}) = v(\{k\})$ and $w(N \setminus \{l\}) > v(N \setminus \{l\})$, we have $\rho_i^w > \rho_i^v$. Because $w(N \setminus \{j\}) = v(N \setminus \{j\})$, $w(\{j\}) = v(\{j\})$,

$w(N)=v(N)$, and $v(N) - v(N \setminus \{j\}) - v(\{j\}) \geq 0$ due to the super-additivity of the game, we have $E_{ij}(w) \geq E_{ij}(v)$ for $i, j \in T$ and $j \neq i$. Thus, $\sum_{j \in T \setminus i} E_{ij}(w) \geq \sum_{j \in T \setminus i} E_{ij}(v)$ for any $i \in T$.

If $i \notin T$, we have $i = l$ since $T = N \setminus \{l\}$. Because $w(\{k\}) = v(\{k\})$, $w(N \setminus \{l\}) > v(N \setminus \{l\})$, and $w(N \setminus \{k\}) = v(N \setminus \{k\})$, for all $k \in N \setminus \{l\}$, we have $\rho_l^w > \rho_l^v$. Moreover, for all $j \in T$, $w(N) = v(N)$, $w(N \setminus \{j\}) = v(N \setminus \{j\})$, $w(j) = v(j)$ and $v(N) - v(N \setminus \{j\}) - v(\{j\}) \geq 0$, we have $E_{lj}(w) \geq E_{lj}(v)$ for all $j \in T$. So $\sum_{j \in T} E_{lj}(w) \geq \sum_{j \in T} E_{lj}(v)$.

To summarize, $\sum_{j \in T} E_{ij}(w) \geq \sum_{j \in T} E_{ij}(v)$ for all $i \in N$. According to Definition 4,
$$\sum_{j \in T} \bar{E}_j(w) = \frac{\sum_{j \in T} \sum_{i \in N} E_{ij}(w)}{n} = \frac{\sum_{i \in N} \sum_{j \in T} E_{ij}(w)}{n} \geq \frac{\sum_{i \in N} \sum_{j \in T} E_{ij}(v)}{n} = \frac{\sum_{j \in T} \sum_{i \in N} E_{ij}(v)}{n} = \sum_{j \in T} \bar{E}_j(v).$$

Subcase 2b: $|T| < |N| - 1$, that is, T contains less than $|N| - 1$ players

For any $i, i \in N$, the self-evaluation value of player i for game w is equal to that for game v , $E_{ii}(w) = E_{ii}(v) = v(N) - v(N \setminus \{i\})$, then we have $\sum_{i \in T} E_{ii}(w) \geq \sum_{i \in T} E_{ii}(v)$.

If T contains more than one player, all peer-evaluation values do not change. For all $i \in N \setminus \{j\}$ and $j \in T$, $w(N \setminus \{k\}) = v(N \setminus \{k\})$ and $w(\{k\}) = v(\{k\})$ for any $k \in N$, we have $E_{ij}(w) = E_{ij}(v)$. According to Definition 4, we have

$$\sum_{j \in T} \bar{E}_j(w) = \frac{\sum_{j \in T} \sum_{i \in N} E_{ij}(w)}{n} = \frac{\sum_{i \in N} \sum_{j \in T} E_{ij}(w)}{n} = \frac{\sum_{i \in N} \sum_{j \in T} E_{ij}(v)}{n} = \frac{\sum_{j \in T} \sum_{i \in N} E_{ij}(v)}{n} = \sum_{j \in T} \bar{E}_j(v).$$

If T only contains a single player, denote the single player by player l , i.e., $T = \{l\}$. We have $\rho_i^w \leq \rho_i^v$ for all $i \neq l$. For $i \notin T$ and $j \notin T$, because $w(N) = v(N)$, $w(N \setminus \{k\}) = v(N \setminus \{k\})$ and $w(\{k\}) = v(\{k\})$ for any $k \notin T$, we have $E_{ij}(w) \leq E_{ij}(v)$ for all $i, j \neq l, i \neq j$. Because $E_{ii}(w) = E_{ii}(v)$ for $i \neq l$ and $w(N) = v(N)$, we have $E_{il}(w) = w(N) - \sum_{j \neq l} E_{ij}(w) - E_{ii}(w) \geq v(N) - \sum_{j \neq l} E_{ij}(v) - E_{ii}(v) = E_{il}(v)$. Then we have $\sum_{j \in N \setminus l} E_{jl}(w) \geq \sum_{j \in N \setminus l} E_{jl}(v)$. According to Definition 4, we have $\sum_{j \in T} \bar{E}_j(w) = \bar{E}_l(w) = \frac{\sum_{i \in N} E_{il}(w)}{n} \geq \frac{\sum_{j \in N} E_{il}(v)}{n} = \bar{E}_l(v) = \sum_{j \in T} \bar{E}_j(v)$.

In conclusion, the proportional cross-evaluation value satisfies weakly coalitional monotonicity.

□

Proof of Proposition 6.

For any $S, T \subset N$ with $S \cap T = \emptyset$, two cases may happen: Case 1: neither S nor T contains Player 1, and Case 2: either S or T contains Player 1. For case 1, we have $c(S) + c(T) = c(S \cup T)$. For case 2, suppose that $1 \in S$ and $T = \{i_1, i_2, \dots, i_k\}$ with $k \geq 1$, then $c(S) + c(T) = c(S) + \sum_{i \in T} c(\{i\}) =$

$$c(S) + c(\{i_1\}) + c(\{i_2\}) + \dots + c(\{i_k\}) \geq c(S \cup \{i_1\}) + c(\{i_2\}) + \dots + c(\{i_k\}) \geq \dots \geq c(S \cup T). \square$$

Proof of Proposition 7.

Consider the game in Figure 2, and take two sub-coalitions of players S and T such that S contains shipper 1 and shipper 2, and T contains carrier 1, shippers 1 and 2. Carrier 1 has Request 1, shipper 1 has Request 2 and shipper 2 has Request 3. $c(S)=30+30+150+150=360$ and $c(T)=70+100+140+30=340$. So we have $c(S)>c(T)$. This implies the SCC cost game considered is not monotone.

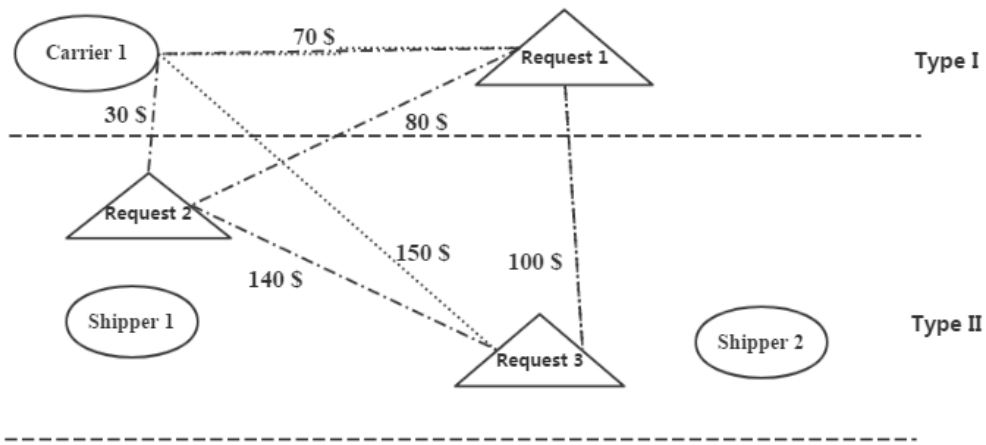


Fig. 2. An example game with one carrier and two shippers □

Proof of Proposition 8.

Consider the game in Figure 3. Take $S = \{\text{Carrier 1}\}$, $T = \{\text{Carrier 1, Shipper 2}\}$, and $k = \text{Shipper 1}$. Carrier 1 has Request 1, shipper 1 has Request 2 and shipper 2 has Request 3.

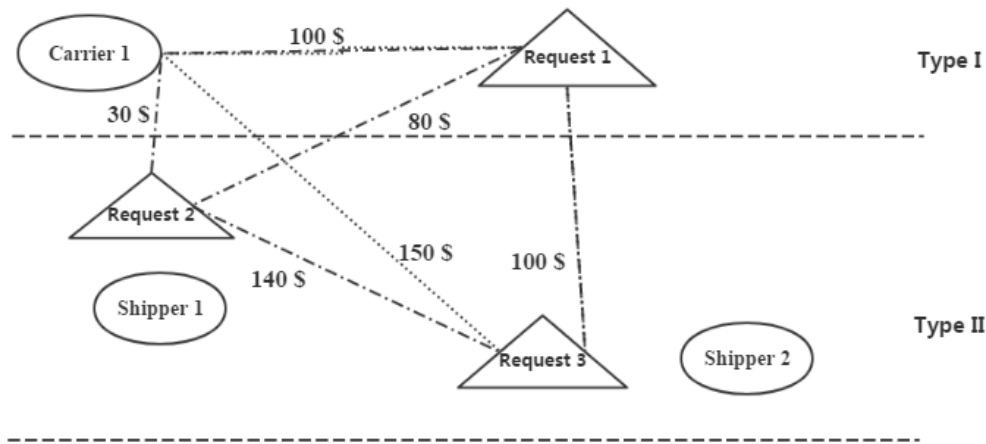


Fig. 3. An example game with one carrier and two shippers

All costs in dollars given in Figure 3 are one-way transportation costs between two locations. According to Figure 3, $c(S)=200$, $c(S \cup \{k\}) = 210$, $c(T)=350$, and $c(T \cup \{k\}) = 370$. $c(S \cup \{k\}) - c(S) = 10$, $c(T \cup \{k\}) - c(T) = 20$. Consequently, $c(S \cup \{k\}) - c(S) < c(T \cup \{k\}) - c(T)$. The SCC cost game (N, c) is thus not convex. \square