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A predictive maintenance policy considering the market price volatility for deteriorating systems

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Abstract

In reality, under the impacts of market dynamism and uncertainty, various input costs of maintenance optimization programs such as spare part cost strongly fluctuate over time. This paper aims at introducing the market price volatility (fluctuation) into the predictive maintenance optimization framework. To this end, we first adopt a Cox–Ingersoll–Ross (CIR) process to describe the evolution of interest rates and a log-diffusion process to model fluctuating maintenance costs. An adapted predictive maintenance policy is then proposed. According to the policy, maintenance decisions are made based on the predictive information about both the system health state and the price volatility level. The consideration of price volatility helps to reduce the negative impacts of the cost fluctuation on the maintenance decision-making, but it makes the evaluation of maintenance performance more complex. To overcome this issue, an analytical solution is developed based on the semi-regenerative properties of the maintained system. Finally, different theoretical and numerical studies confirm the necessity of taking into account the price volatility and the effectiveness of the proposed policy.

Keywords: Predictive maintenance; Price volatility; Cox–Ingersoll–Ross model; Log-diffusion process; Gamma process; Semi-regenerative process; Conditional reliability.

1. Introduction

Predictive maintenance (PdM), one of the most modern maintenance techniques, uses the prognostic information about the future system health to determine when maintenance actions are necessary [1]. The predictive information about the system health state such as Remaining Useful Life (RUL), Mean Residual Life (MRL), Conditional Reliability (CRe), future degradation level/production rate, etc., can be estimated from the condition monitoring data (e.g. vibration, acoustic emission, temperature data, . . .) by use of industrial prognostic techniques. If the estimation is reliable and the PdM is properly implemented, it can help to significantly reduce the maintenance cost and improve the system availability [2].

Given many advantages, the PdM has been extensively developed by several authors. The existing works on PdM may be categorized into the two following topics : (a) data acquisition and processing; (b) maintenance decision-making [3]. In the first topic, different techniques and algorithms have been proposed to collect the monitoring data and to predict the system health evolution based on the collected data. Among them, the Internet of Things (IoT) sensors and machine learning algorithms have achieved great success and received widespread attention, especially in the industry 4.0 context [4, 5, 6, 7]. Meanwhile, for the second topic, the main aim is to develop PdM policies to optimize the predictive maintenance decision-making. Interested in the second topic, we consider hereinafter two kinds of PdM policies: control-limit and cost-balancing. According to the control-limit-based policies, the system is maintained at an inspection time if the predicted system health state at this time reaches a critical threshold. Huynh et al. [8] proposed a PdM policy in which the system is inspected regularly. At an inspection time, the MRL is predicted and used as an index for the maintenance decision making. The system is replaced if the predicted MRL

is smaller than a specific limit. The authors underlined that the decision making based on the MRL is more efficient than that based on the degradation level because the MRL indicator contains more information on the system state compared to the degradation level. Vu et al. [9] developed an adapted MRL-based model for multi-component systems with redundancy and economic dependence between their components (the joint maintenance of several components reduces the maintenance cost). According to the proposed policy, the replacement decision of a component is made in terms of its MRL as well as its important role in the system functioning represented by the Birnbaum importance measure. The conditional reliability (CRe), the probability that a functioning system can survive a certain amount of time, was also used as a PdM decision indicator in literature. Huynh et al. [10] developed a CRe-based model for k-out-of-n systems. Nguyen et al. [11, 12] proposed an adapted CRe-based model with taking into account the economic dependence and spare part provisioning. In addition to the MRL and CRe, the RUL has been largely and recently used for PdM decision-making. For more detail, Chen et al. [13] and Nguyen et al. [14] proposed to replace the system whenever its RUL (remaining time before system failure) at an inspection is smaller than a specific limit. Do et al. [15] and Chen et al. [16] developed the same RUL-based policy with consideration of the imperfect maintenance (the system is somewhere between “as bad as old” state and “as good as new” state after imperfect maintenance). Omshi et al. [17] used the RUL-based limit to decide the next inspection time.

The second kind of PdM policies (cost-balancing-based policies) is more complicated than the first one. Indeed, according to this kind, the maintenance decisions are done by comparing the expected costs of particular maintenance options. In [18], the maintenance decision is made based on the comparison between the expected cost for “preventive maintenance” (PM) and “no pre-

ventive maintenance” at each inspection time. The authors considered the risk of system failure in the next operational period and productivity loss due to system degradation. Shi et al. [19] considered the same options (PM and no PM) for multi-component systems with stochastic dependence (the failure of a component influences the degradation of the others). Lei et al. [20] developed a PdM policy for wind farms. The decision is made by balancing between the risk of system failure and the portion of the RUL thrown away. Finally, the authors in [21, 22, 23, 24] developed grouping PdM policies for multi-component systems with economic dependence. The PM is planned for a short-term interval instead of only at inspection time. The best groups of PM activities are found by maximizing the grouping economic profit in the interval. Finally, with the development of Industry 4.0, the perspective maintenance approach using machine learning, semantic reasoning, and simulation methods has been recently developed [25, 26, 27]. According to this approach, the maintenance decision is made by balancing many factors including not only the maintenance costs but also the production planning, the spare part logistic. In addition, the maintenance knowledge-base is also integrated in the decision-making.

All these above PdM policies were developed by assuming that the maintenance costs such as the price of spare parts (used for replacement of degraded/failed system) are constants. Van der Weide et al. [28] proposed a maintenance cost model which takes into account the discounted cash flow. In this work, the price of spare parts and maintenance cost in general are considered to be discounted over time with an exponential discounting factor, i.e. $c(t_k) = c(t_{k-1}) \cdot e^{-\Delta t \cdot r}$, where r is the constant discount rate and $\Delta t = t_k - t_{k-1}$. Nguyen et al. [29] developed a maintenance model under technological change. The authors assumed that the price of spare parts is decreasing over time after appearance and normally increasing over technological generation. However, in

reality, the market price is naturally dynamic and uncertain due to many factors such as product cost, utility and demand, the extent of competition, government and legal regulations, or even unexpected events such as natural disasters, etc.
85 The maintenance/spare part cost may be therefore randomly fluctuating over time. The maintenance cost models proposed in the previous works therefore do not fully capture the dynamism and uncertainty of the market price. In addition, the consideration of the price volatility/fluctuation is only limited to the maintenance modeling level.

90 The price volatility and its impacts have been extensively studied in oil and gas industries [30], food industry [31], electricity market [32], etc; however, according to our best knowledge, they have not been considered yet in the maintenance decision-making. In addition, no specific maintenance decision rule has been proposed for this situation in literature. The aims of this paper
95 are threefold :

- Identify an indicator that allows to represent the price volatility; introduce the identified indicator to the PdM modeling;
- Develop an adapted PdM policy that allows to reduce the negative impacts of the price volatility on the PdM decision-making;
- 100 • Propose analytical methods to estimate the PdM performance based on semi-regenerative property of the maintained system.

The remainder of this paper is organized as follows: Section 2 presents some general assumptions and the PdM modeling; an adapted PdM policy is proposed in Section 3; the stochastic characteristics of the maintained system as well as
105 the performance of the proposed policy are analyzed in Sections 4 and 5; the effectiveness of our policy is verified by numerical examples in Section 6; finally, Section 7 reports some conclusions drawn from this work.

List of Notations and Abbreviations

PdM	Predictive Maintenance	r_k	Interest rate value at t_k
PR	Preventive replacement	r_0	Interest rate value at $t = 0$
CR	Corrective replacement	$f_{r_t r_s}(u v)$	Transition density function of r_t
CIR	Cox-Ingersoll-Ross process	$P_{C^p_{t_k+1} C^p_{t_k}}(z z)$	PR cost-saving probability at t_k
LD	Log-Diffusion process	P_0	PR cost-saving threshold
p.d.f	Probability Density Function	P_0^*	Optimal PR cost-saving threshold
w.r.t	With Respect To	$I_Q(\cdot)$	Modified Bessel function
t_k	Time of k^{th} inspection	$Q_m(p, q)$	Marcum-Q function
ΔT	Inspection cycle	a	Mean reversion speed
ΔT^*	Optimal inspection cycle	b	Long-run mean
$(X_t)_{t \in \mathbb{R}^+}$	Degradation process	c	Volatility rate
$(\tilde{X}_t)_{t \in \mathbb{R}^+}$	Degradation of the maintained system	$(C_t^p)_{t \in \mathbb{R}^+}$	PR cost process
$(\tilde{X}_k)_{k \in \mathbb{N}}$	Markov chain embedded in $(\tilde{X}_t)_{t \in \mathbb{R}^+}$	$(C_t^c)_{t \in \mathbb{R}^+}$	CR cost process
L	Failure threshold	ϵ_p	PR constant
α	Gamma shape parameter	ϵ_c	CR constant
β	Gamma scale parameter	π_2	Stationary law of \hat{r}_k
$f_{\alpha, \beta}(x)$	p.d.f of gamma distribution	π_1	Conditional stationary law of \tilde{X}_k
$R_{t_f X_{t_k}}(t x)$	System conditional reliability at t_k	π	Joint stationary law of \hat{Y}_k
R_0	Reliability threshold	c_i	Inspection cost
R_0^*	Optimal reliability threshold	c_d	Downtime cost rate
$\Gamma(y)$	Gamma function	C_∞	Long-term maintenance cost rate
$\gamma(\alpha, y)$	Lower incomplete gamma function	$C_p(T)$	Cumulative PR cost
$(r_t)_{t \in \mathbb{R}^+}$	Interest rate process	$C_c(T)$	Cumulative CR cost
$(\tilde{r}_k)_{k \in \mathbb{N}}$	Markov chain embedded in $(r_t)_{t \in \mathbb{R}^+}$	$\tau(T)$	Cumulative operational time
$(Y_t)_{t \in \mathbb{R}^+}$	Joint process of $(\tilde{X}_t)_{t \in \mathbb{R}^+}$ and $(r_t)_{t \in \mathbb{R}^+}$	$\mathbb{E}_\pi(\cdot)$	Expected values w.r.t measure π
$(\hat{Y}_k)_{k \in \mathbb{N}}$	Couple embedded Markov Chain		

2. PdM modeling under the market price volatility

110 This section is devoted to some general assumptions and describes how maintenance cost and system deterioration can be modeled by using stochastic models.

2.1. General description

Consider a new system put into operation at time $t_0 = 0$. As a result
115 of thermal and mechanical stresses, impurities, chemical corrosion, etc., the system deteriorates over time. The deterioration process is complex and depends on many uncertain factors such as temperature, humidity, or workload. It is then risky to describe the degradation level of the system at instant t by a deterministic variable. For this reason, the degradation level at a given time
120 is usually considered as a random variable that follows a specific probability

distribution [33]. The intrinsic degradation evolution can be then presented by a continuous stochastic process, denoted by $(X_t)_{t \in \mathbb{R}^+}$ with $X_0 = 0$. Moreover, it should be positive and strictly increasing with time to represent the fact that the system worsens due to aging and accumulated wear or damage [34].

125 The assessment of the degradation level can be done continuously in real time by a continuous monitoring system or occasionally at specific instants by inspections. For example, the vibration level of a machine can be monitored continuously by vibration sensors; otherwise, it is hard to monitor the crack size that appears inside the mechanical components or concretes continuously. 130 In reality, the crack size is measured by an inspection team at specific instants [35]. In this paper, the second option is considered. As such, the degradation level of the system X_{t_k} is assessed only at inspection times t_k , $k \in \mathbb{N}$. The inspection is considered to be: periodic (the inspection is done at every ΔT time units, i.e., $t_k = t_{k-1} + \Delta T$); perfect (the inspection can detect the exact 135 degradation level); non-destructive (the inspection does not affect the system state and its degradation level). An inspection incurs a constant cost c_i .

Without any maintenance action, the system degradation level will increase over time. The system is considered to be failed when its degradation level reaches a critical threshold, which is defined according to some economic or 140 safety specifications. The system failures are not self-announcing. They are therefore detected only at inspection times. In case of failure between two consecutive inspection times, the system is then unavailable from its failure time to the next inspection time. During the system failure, a downtime cost rate c_d has to be paid every time unit. We assume that the system does not 145 deteriorate when it does not work.

At an inspection time t_k , two following types of maintenance actions are considered: Corrective Replacement (CR) and Preventive Replacement (PR).

The CR is carried out to restore the system in operation if it failed, i.e., $X_{t_k} \geq L$; Otherwise, the PR is planned when the degradation level X_{t_k} has not yet reached its critical limit to prevent the system from failures. After either a CR or PR,
150 the degradation level of the system is reset to zero. The CR and PR durations are neglected when compared to the duration of the planning horizon. We also assume that the maintenance support (spare part, repair team) is sufficient and always available to guarantee that these replacement actions can be done at any
155 time.

As mentioned in the introduction section, under the market price volatility, the spare part cost, which is an important part of any replacement project, fluctuates over time. To capture the fluctuations, stochastic processes are used to model the CR and PR costs. Let $(C_t^c)_{t \in \mathbb{R}^+}$ and $(C_t^p)_{t \in \mathbb{R}^+}$ denote the two
160 continuous stochastic processes that describe the evolution of CR and PR costs over time. These two processes have the following properties: (a) C_t^c is more expensive than C_t^p at any time t ; (b) $(C_t^c)_{t \in \mathbb{R}^+}$ and $(C_t^p)_{t \in \mathbb{R}^+}$ may increase or decrease over time.

In the next subsections, we will present in more detail the stochastic models that are selected to model the degradation and replacement cost processes.
165

2.2. Degradation model

In our work, the gamma process is selected to model the system deterioration. This process is strongly recommended for the modeling of monotonic and gradual deterioration [36]. It has been successfully applied to many appli-
170 cations with real degradation data such as creep of concrete data [37], fatigue crack growth data [38], thinning due to corrosion data [39].

The degradation process $(X_t)_{t \in \mathbb{R}^+}$ is assumed to be described by a homogeneous gamma stochastic process. For more detail, degradation increments between non-overlapping intervals are mutually and stochastically independent.

The degradation increment $X = X_\omega - X_t$ between t and ω ($t \leq \omega$) follows a gamma distribution with probability density function (p.d.f):

$$f_{\alpha \cdot (w-t), \beta}(x) = \frac{1}{\Gamma(\alpha \cdot (w-t))} \cdot \beta^{\alpha \cdot (w-t)} \cdot x^{\alpha \cdot (w-t)-1} \cdot e^{-\beta x} \cdot 1_{\{x>0\}} \quad (1)$$

175 where, $\Gamma(y) = \int_0^{+\infty} u^{y-1} e^{-u} du$ denotes the complete gamma function; α and β are the shape and scale parameters respectively; the average and variance values of the increment are $\mu = \alpha/\beta$ and $\sigma = \alpha/\beta^2$. The behavior of the degradation process then depends closely on the couple of parameters (α, β) .

Thanks to equation 1, some following properties of the deterioration modeled
180 by gamma process can be deduced: (*P1*) the increment distribution depends only on the length of the considered interval $(w - t)$. It does not depend on the historical degradation levels before ω ; (*P2*) the increments between periodic inspection periods are then *i.i.d* (independent and identically distributed); (*P3*) given that the increments are positive, the degradation process is then positive
185 and strictly increasing with time. While property *P3* is verified for most of the real applications such as crack size propagation and energy consumption, the validation of *P1* and *P2* is not always guaranteed in reality since the future degradation speed may depend on the current/historical degradation levels.

2.3. Stochastic replacement cost model

Stochastic interest rate. To integrate the market price volatility into the CR and PR costs processes, we model first the interest rate, one of the key variables in the economy, driving the asset, bond price evolution. For this purpose, the Cox-Ingersoll-Ross (CIR) process, a diffusion process introduced by [40], is selected. This is one of the most employed interest rate models in literature thanks to

its interesting features such as the non-negativity, the mean reversion¹, and the relative tractability. The CIR process, denoted by $(r_t)_{t \in \mathbb{R}^+}$, is the solution of the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + c\sqrt{r_t}dW_t \quad (2)$$

190 where, a, b, c are positive constant parameters representing the mean reversion speed, the long-run mean, and the volatility rate respectively; $(W_t)_{t \in \mathbb{R}^+}$ is a standard Brownian Motion. It has the following properties: (P1) $W_0 = 0$; (P2) stationary and independent increments: $W_t - W_s \sim \mathcal{N}(0, t - s)$ for $0 < s < t$.

By using the decomposition of Bessel bridges, [41] shown that the solution of equation 2 is a process with the following transition density function:

$$f_{r_t|r_s}(u | v) = Ae^{-(B+C)} \left(\frac{C}{B}\right)^{\frac{Q}{2}} I_Q(2\sqrt{BC}); t > s \geq 0 \quad (3)$$

where,

$$A = \frac{2a}{c^2 [1 - e^{-a(t-s)}]}; \quad B = A \cdot v \cdot e^{-a(t-s)}; \quad C = A \cdot u; \quad Q = \frac{2ab}{c^2} - 1; \quad (4)$$

and $I_Q(\cdot)$ is the modified Bessel function of the first kind of order Q :

$$I_Q(x) = \sum_{j=0}^{\infty} \left(\frac{x}{2}\right)^{2j+Q} \frac{1}{j!\Gamma(j+Q+1)} \quad (5)$$

The mean and variance of r_t given $r_s = v$ are

$$\mathbb{E}[r_t | r_s] = v \cdot e^{-a(t-s)} + b \cdot (1 - e^{-a(t-s)}); \quad \lim_{t \rightarrow +\infty} \mathbb{E}[r_t | r_s] = b \quad (6)$$

¹Mean reversion in finance suggests that asset prices and historical returns eventually revert to their long-term mean or average level (<https://www.investopedia.com>).

and

$$\text{Var}[r_t | r_s] = v \cdot \frac{c^2}{a} \left[e^{-a(t-s)} - e^{-2a(t-s)} \right] + \frac{bc^2}{2a} \left[1 - e^{-a(t-s)} \right]^2; \quad \lim_{t \rightarrow +\infty} \text{Var}[r_t | r_s] = \frac{bc^2}{2a}. \quad (7)$$

If $d = \frac{4ab}{c^2}$ is an integer, $f_{r_t|r_s}(u | v)$ can be linked to the p.d.f of the non-central chi-square distribution. Indeed, by replacing $B = \frac{\lambda}{2}; U = 2Au \rightarrow C = Au = \frac{U}{2}$; and $Q = \frac{d}{2} - 1$ into equation 3 [42], we have

$$f_{r_t|r_s}(u | v) = 2A \cdot \left[\frac{1}{2} e^{-\frac{\lambda+U}{2}} \left(\frac{U}{\lambda} \right)^{\frac{d}{4}-\frac{1}{2}} I_{\frac{d}{2}-1}(\sqrt{\lambda U}) \right] = 2A \cdot g(U, d, \lambda) \quad (8)$$

where, $g(U, d, \lambda)$ is the p.d.f of the non-central chi-square distribution with the
195 degrees of freedom d and non-centrality parameter λ (see Appendix A).

When compared to the system deterioration modeled by gamma process, the
interest rate modeled by the diffusion CIR process is not increasing in time. In
addition, equation 3 shows that the interest rate at time t depends not only on
the length $t-s$, but also its value at time t . Figure 1a sketches an example of the
200 interest rate modeled by a CIR process with $a = 0.4360; b = 0.0612; c = 0.1633$;
and $r_0 = 0.08$ (interest rate level at time $t = 0$).

Stochastic replacement costs. Based on the stochastic interest rate model, the
PR and CR costs can be modeled. In the paper, the most simple model, named
log-diffusion (LD) process, is adapted as follows [43]:

$$\begin{cases} C_t^p = \epsilon_p \cdot e^{r_t} \\ C_t^c = \frac{\epsilon_c}{\epsilon_p} \cdot C_t^p \end{cases} \quad (9)$$

where, $\epsilon_p > 0$ and $\epsilon_c > 0$ are PR and CR constants. When compared to the
PR, the CR can not be planned and its cost may be composed of the different
costs associated with failures such as system damages, negative environmental

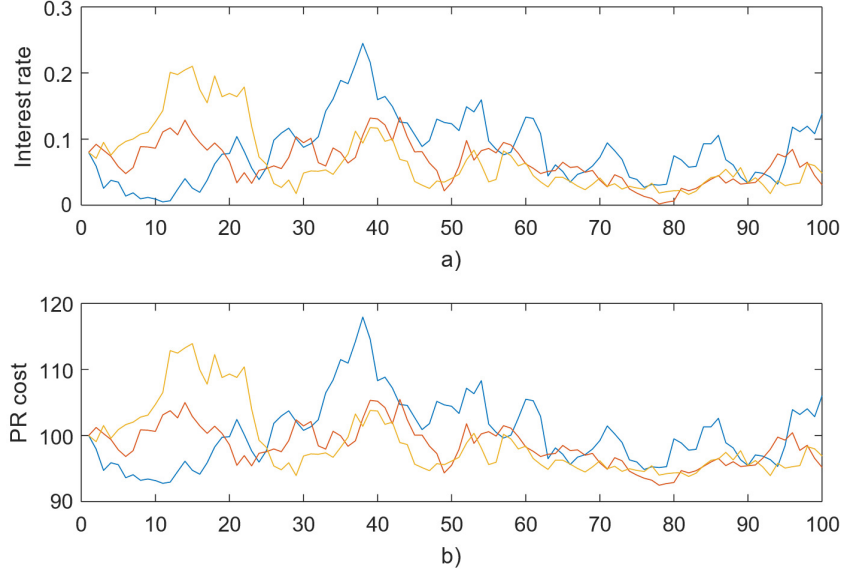


Figure 1: (a) three interest rate paths modeled by the CIR model; (b) three PR cost paths modeled by the LD model

205 impacts, human safety issues. A CR is then more expensive than a PR, i.e., $\epsilon_c > \epsilon_p$. Figure 1b sketches an example of the PR cost modeled by the LD process with $\epsilon_p = 92.3116$. Finally, it should be noted that the LD model is chosen to simplify further investigations of the paper. The other price/cost models such as double-diffusion process [44] or auto-regressive process (ARCH, 210 GARCH) [45] may be considered in the future research.

3. Description of the proposed PdM policy

Since the replacement costs vary over time, it is important to plan the replacement project at the right time (in low-cost periods). For this reason, in our paper, the replacement decisions are made based on not only the pre- 215 dictive information about the system health state but also about the interest rate/replacement costs. In this section, we will present first the two maintenance decision basis before going into more detail on the description of our PdM pol-

icy. Finally, to evaluate the performance of the proposed model, a cost-based criterion is introduced.

220 3.1. Maintenance decision indicators

System conditional reliability. The system conditional reliability is used to represent the system health state in our study. It is a popular maintenance decision indicator. The reliability is defined as the probability that the system does not fail before time t given that it is functioning at time s and $X_s = x$. To calculate the system conditional reliability, let t_f denote the system failure time,

$$t_f = \inf \{t \in \mathbb{R}^+ \mid X_t \geq L\} \quad (10)$$

The probability that the system fails before time t is calculated as [36]

$$\begin{aligned} F_{t_f|X_s}(t \mid x) &= \mathbb{P}(t_f \leq t \mid X_t = x) = \mathbb{P}(X_t \geq L \mid X_s = x) = \mathbb{P}(X_t - X_s \geq L - x) \\ &= \frac{\Gamma(\alpha(t-s), \beta(L-x))}{\Gamma(\alpha(t-s))} \cdot 1_{\{x < L\}} + 1_{\{x \geq L\}} \end{aligned} \quad (11)$$

where $\Gamma(\alpha, x) = \int_x^{+\infty} u^{\alpha-1} e^{-u} du$ is the upper incomplete gamma function. The system conditional reliability, denoted by $R_{t_f|X_s}(t \mid x)$, is then

$$R_{t_f|X_s}(t \mid x) = 1 - F_{t_f|X_s}(t \mid x) = 1 - \frac{\Gamma(\alpha(t-s), \beta(L-x))}{\Gamma(\alpha(t-s))} = \frac{\gamma(\alpha(t-s), \beta(L-x))}{\Gamma(\alpha(t-s))} \quad (12)$$

where $\gamma(\alpha, x) = \int_0^x u^{\alpha-1} e^{-u} du$ is the lower incomplete gamma function, and $\gamma(\alpha, x) + \Gamma(\alpha, x) = \Gamma(\alpha)$. Thanks to the above equation, given that the system degradation is x at inspection time t_k , the reliability of the system at the next inspection time $t_{k+1} = t_k + \Delta T$ can be calculated as

$$R_{t_f|X_{t_k}}(t_{k+1} \mid x) = \frac{\gamma(\alpha \Delta T, \beta(L-x))}{\Gamma(\alpha \Delta T)} \quad (13)$$

A low value of $R_{t_f|X_{t_k}}(t_{k+1} | x)$ means that the probability that the system fails during the next operation period is high. From a reliability point of view, the PR at t_k is necessary to avoid the system failures.

PR cost-saving probability. The PR cost-saving probability $P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z)$ is defined as the probability that the PR cost at time t_{k+1} is cheaper than that at t_k .

$$\begin{aligned} P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z) &= \mathbb{P}\left(C_{t_{k+1}}^p < z \mid C_{t_k}^p = z\right) = \mathbb{P}\left(\epsilon_p e^{r_{t_{k+1}}} < z \mid \epsilon_p e^{r_{t_k}} = z\right) \\ &= \mathbb{P}\left(r_{t_{k+1}} < Z \mid r_{t_k} = Z\right) = \int_0^Z f_{r_{t_{k+1}}|r_{t_k}}(u | Z) du \end{aligned} \quad (14)$$

where, $Z = \ln(z/\epsilon_p)$; and $f_{r_{t_{k+1}}|r_{t_k}}(u | Z)$ is calculated by equation 3. If d is an integer, $P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z)$ can be rewritten as

$$\begin{aligned} P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z) &= \int_0^Z 2A \cdot g(U, d, \lambda) du = 1 - \int_{2AZ}^{+\infty} g(U, d, \lambda) dU \\ &= 1 - \int_{2AZ}^{+\infty} \frac{1}{2} e^{-\frac{\lambda+U}{2}} \left(\frac{U}{\lambda}\right)^{\frac{d}{4}-\frac{1}{2}} I_{\frac{d}{2}-1}(\sqrt{\lambda U}) dU \end{aligned} \quad (15)$$

By replacing $t = \sqrt{U}$; $p = \sqrt{\lambda}$; $m = \frac{d}{2}$; and $q = \sqrt{2AZ}$, we finally obtain

$$P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z) = 1 - Q_{\frac{d}{2}}(\sqrt{\lambda}, \sqrt{2AZ}) = 1 - Q_{\frac{2ab}{c^2}}\left(\sqrt{2AZ} \cdot e^{-\frac{a\Delta T}{2}}, \sqrt{2AZ}\right) \quad (16)$$

where, $Q_m(p, q)$ is the Marcum-Q-function, i.e.

$$Q_m(p, q) = \frac{1}{p^{m-1}} \int_q^{+\infty} t^m e^{-\frac{t^2+p^2}{2}} I_{m-1}(pt) dt \quad (17)$$

It is important to note that equation 16 is used to calculate the PR cost-saving probability only when m is an integer number; otherwise, equation 14 should be used in other cases. When $P_{C_{t_{k+1}}^p|C_{t_k}^p}(z | z)$ is high, the replacement at

t_{k+1} is probably cheaper than that at t_k . From an economic point of view, it is preferred to do the PR at t_{k+1} than at t_k . Finally, CR is directly triggered by system failures. The CR is therefore can not be planned. CR cost-saving probability is then not interesting for maintenance decision-making, and not considered here.

3.2. PdM decision rules

In our maintenance model, a PR decision at each inspection time is done regarding both the system health state (system conditional reliability) and the price volatility level (PR-cost saving probability). For more detail, assume that the degradation level and the interest level at inspection time t_k are $X_{t_k} = x$ and $r_{t_k} = z$ respectively. The following maintenance rules are adopted:

- If $x \geq L$, i.e. the system failed at t_k , a CR activity should be carried out immediately to restore it to the new state;
- If $x < L$ and $R_{t_f|X_{t_k}}(t_{k+1} | x) \leq R_0$, a PR at t_k is a promising solution to improve the system reliability. However, from an economic point of view, if $P_{C_{t_{k+1}}^p} | C_{t_k}^p(z | z)$ is high, i.e., the PR at t_k is probably more expensive than that at t_{k+1} . It is therefore more reasonable to postpone and reconsider the PR at time t_{k+1} . In summary, two following scenarios are possibles:
 - if $x < L, R_{t_f|X_{t_k}}(t_{k+1} | x) \leq R_0, P_{C_{t_{k+1}}^p | C_{t_k}^p}(z | z) \leq P_0$, a PR activity is carried out at t_k ;
 - if $x < L, R_{t_f|X_{t_k}}(t_{k+1} | x) \leq R_0, P_{C_{t_{k+1}}^p | C_{t_k}^p}(z | z) > P_0$, the system is left unchanged at t_k
- If $x < L$ and $R_{t_f|X_{t_k}}(t_{k+1} | x) > R_0$, the system is left unchanged at t_k .

The inspection cycle ΔT , R_0 , and P_0 are the parameters of our maintenance policy. They will be optimized with respect to the priority goal of the maintenance program (see the next section for more details).

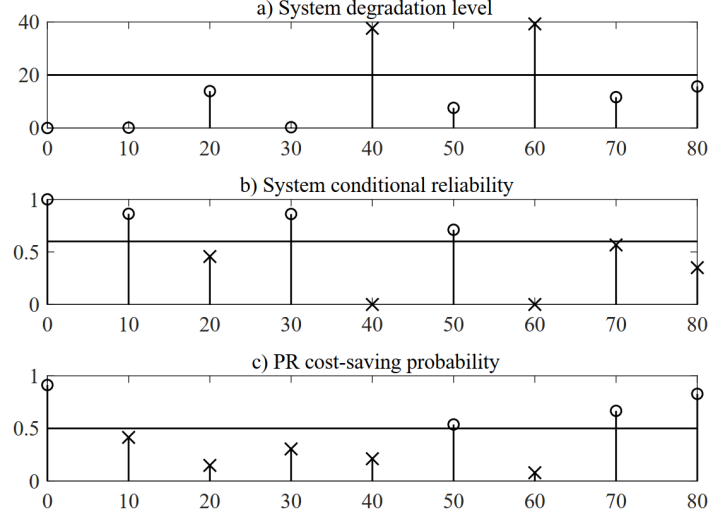


Figure 2: Illustration of the maintenance decision-making when the proposed PdM policy is applied

Figure 2 shows the evolution of the system degradation (figure 2a), system
 255 conditional reliability (figure 2b) and PR cost-saving probability (figure 2c) in
 interval $[0, 80]$ when the proposed policy is applied with the following parameters: $\Delta T = 10$, $R_0 = 0.6$, and $P_0 = 0.5$. At $t_1 = 10$, the system is left as it is
 because $X_{t_1} < L = 20$ and $R_{t_f|X_{t_1}}(t_2 | x) > R_0$. At time $t_2 = 20$, $X_{t_2} < L$,
 $R_{t_f|X_{t_1}}(t_2 | x) < R_0$, and the probability that the PR cost at t_3 is cheaper than
 260 that at t_2 is small ($P_{C_{t_3}^p|C_{t_2}^p}(z | z) < P_0$), a PR activity is then carried out immediately at this time. Otherwise, at time $t_7 = 70$, $X_{t_7} < L$, $R_{t_f|X_{t_7}}(t_8 | x) < R_0$,
 but the probability that the PR cost at t_8 is cheaper than that at t_7 is high
 ($P_{C_{t_8}^p|C_{t_7}^p}(z | z) > P_0$), PR is then not planned at t_7 . CR activity is carried out
 at $t_4 = 40$ and at $t_6 = 60$ because $X_{t_4} > L$ and $X_{t_6} > L$.

In the literature, the quantities such as RAMS (Reliability, Availability, Maintainability, Safety) or maintenance cost are usually used to assess the performance of a maintenance model [46]. The most appropriate criteria are selected regarding the specific system requirements. For example, the reliability and safety are the priority objectives of the maintenance program of safety-critical systems [47]; otherwise, for the production systems, the maintenance cost and/or system availability are usually considered as the maintenance objectives [48]. In our paper, long-run expected maintenance cost rate, the most popular maintenance objective in the literature, is used to measure the performance of the proposed model. The long-run expected maintenance cost rate is defined as

$$C_{\infty}(\Delta T, R_0, P_0) = \lim_{t \rightarrow \infty} \frac{\mathbb{E}_0[C(t)]}{\mathbb{E}_0[\tau(t)]} \quad (18)$$

where, $\tau(t)$ is the cumulative operational time of the system until t ; $C(\tau)$ is the cumulative maintenance cost; and $\mathbb{E}_0[\cdot]$ is the expected values given that the degradation at time $t = 0$ is zero.

By the definition, C_{∞} allows reflecting at the same time the maintenance cost $C(t)$ and the system availability $\tau(t)$. The estimation of this cost criterion is a complicated problem and mostly done by stochastic simulation methods [49]. The analytical methods are possible but limited to some classes of the maintenance models with some specific stochastic properties. For this reason, in the next section, we will discuss about the stochastic behaviors of the system when the proposed maintenance policy is applied.

4. Stochastic properties of the maintained system

The analytical calculation of $C_{\infty}(\Delta T, R_0, P_0)$ requires the derivation of stochastic characteristics of the maintained system. We analyze therefore the

characteristics of the two following stochastic processes, which conduct the
 280 stochastic behavior of our model: (a) degradation process of the maintained
 system denoted by $(\tilde{X}_t)_{t \in \mathbb{R}^+}$; (b) interest rate process $(r_t)_{t \in \mathbb{R}^+}$.

4.1. Degradation and interest rate joint process

According to our maintenance model, the PR decisions are made based on
 both the system conditional reliability and the PR cost (Subsection 3.2). The
 285 degradation process of the maintained system $(\tilde{X}_t)_{t \in \mathbb{R}^+}$ therefore depends on
 the interest rate process $(r_t)_{t \in \mathbb{R}^+}$. It means that the two processes should be
 considered as a joint process, denoted by $(Y_t)_{t \in \mathbb{R}^+} = (\tilde{X}_t, r_t)_{t \in \mathbb{R}^+}$.

According to [50], $(Y_t)_{t \in \mathbb{R}^+}$ is a semi-regenerative process with the semi-
 regenerative points equal the inspection times $t_k (k \in \mathbb{N})$. Indeed, the evolution
 290 of the joint process $(Y_t)_{t \in \mathbb{R}^+}$ between two consecutive inspection times t_k and
 t_{k+1} depends only on its current values at the beginning of t_k , and does not
 depend on their past values according to equations 3 and 11. In other words,
 conditional on the values of $(Y_t)_{t \in \mathbb{R}^+}$ at the beginning of t_k , its evolution be-
 tween t_k and t_{k+1} can be completely determined. The discrete-time process
 295 describing the state of Y_t at the beginning of each inspection time is a couple
 embedded Markov Chain (MC), denoted $(\hat{Y}_k)_{k \in \mathbb{N}} = (\hat{X}_k, \hat{r}_k)_{k \in \mathbb{N}}$, with contin-
 uous state space in $(\mathbb{R}^+ \times \mathbb{R}^+)$. $(\hat{X}_k)_{k \in \mathbb{N}}$ and $(\hat{r}_k)_{k \in \mathbb{N}}$ are the single embedded
 Markov Chain of \tilde{X}_t and r_t at the beginning of each inspection time respectively.
 Thanks to the semi-regenerative property, an analysis of the system behavior on
 300 a single semi-regenerative cycle delimited by two consecutive inspection times
 can be considered for the calculation of the long-run expected maintenance cost
 rate. The calculation is therefore simplified because the number of possible
 maintenance scenarios that have to be considered is strongly reduced. The
 price to pay for this simplified analysis is the derivation of the stationary law
 305 of $(\hat{Y}_k)_{k \in \mathbb{N}}$ [51]. In the next subsection, we will focus on the calculation of

this stationary distribution, which is the basis for the further analysis of the long-run expected maintenance cost rate.

4.2. Stationary distribution of $\hat{\mathbf{Y}}_{\mathbf{k} \in \mathbb{N}}$

Invariant equation. The stationary law of $\hat{Y}_k = (\hat{X}_k, \hat{r}_k)$, denoted by π , is defined as the solution of the following invariant equation:

$$\pi(x, u) = \int_0^\infty \int_0^\infty \pi(y, v) \cdot p(x, u \mid y, v) dy dv \quad (19)$$

where, $p(x, u \mid y, v)$ is the transition probability density function from $\hat{Y}_k = (y, v)$ at the current inspection time t_k to $\hat{Y}_{k+1} = (x, u)$ at t_{k+1} . Moreover, according to the definition of conditional probability, we have

$$\pi(y, v) = \pi_1(y \mid v) \cdot \pi_2(v) \quad (20)$$

where,

- 310 • π_1 is the stationary law of the single embedded MC $(\hat{X}_k)_{k \in \mathbb{N}}$ given a certain level of the interest rate;
- π_2 is the stationary law of the single embedded MC $(\hat{r}_k)_{k \in \mathbb{N}}$. Jin et al. [52] shown that π_2 is a gamma law of the following form:

$$\pi_2(v) = \frac{1}{\Gamma(\alpha_r)} \cdot \beta_r^{\alpha_r} \cdot v^{\alpha_r-1} \cdot e^{-\beta_r v} \cdot 1_{\{v \geq 0\}} \quad (21)$$

where, $\alpha_r = \frac{2ab}{c^2} = \frac{d}{2}$ and $\beta_r = \frac{2a}{c^2} = \frac{\alpha_r}{b}$.

Thanks to equations 20 and 21, π can be calculated if π_1 is known. π_1 is the solution of the following invariant equation:

$$\pi_1(x \mid u) \cdot \pi_2(u) = \int_0^\infty \int_0^\infty \pi_1(y \mid v) \cdot \pi_2(v) \cdot p(x, u \mid y, v) dy dv \quad (22)$$

This equation is deduced from equations 19 and 20.

Transition probability density function $p(x, u \mid y, v)$. To solve equation 22, we first calculate $p(x, u \mid y, v)$

$$p(x, u \mid y, v) = p(x \mid u, y, v) \cdot p(u \mid y, v) = p(x \mid y, v) \cdot p(u \mid v) \quad (23)$$

we have

- 315 • $p(u \mid y, v) = p(u \mid v)$ due to the fact that the degradation level of the system has no impact on the evolution of the interest rate process. $p(u \mid v)$ is equal to $f_{r_t|r_a}(u \mid v)$ and can be calculated by using equation 3;
- $p(x \mid u, y, v) = p(x \mid y, v)$, since the degradation level of the maintained system at t_{k+1} does not depend on the value of the interest rate at this
- 320 time.

In order to calculate $p(x \mid y, v)$, all possible maintenance scenarios in a semi-regenerative cycle (time interval between two consecutive inspections) should be analyzed to understand the transition of $\left(\widehat{X}_k\right)_{k \in \mathbb{N}}$ from $\widehat{X}_k = y$ at t_k to $\widehat{X}_{k+1} = x$ at t_{k+1} given $\widehat{r}_k = v$

Finally, it should be noted that $R_{t_f|X_{t_k}}(t_{k+1} \mid y)$ is a monotonically decreasing function of x ; and $P_{C_{t_{k+1}}^p|C_{t_k}^p}(v \mid v)$ is a monotonically increasing function of v at the inspection time t_k [53]. We have therefore

$$R_{t_f|X_{t_k}}(t_{k+1} \mid x) < R_0 \Leftrightarrow y > x^0 \text{ with } x^0 = R^{-1}(R_0) \quad (24)$$

and

$$P_{C_{k+1}|C_k}(z \mid z) > P_0 \Leftrightarrow v > r^0 \text{ with } r^0 = P^{-1}(P_0) \quad (25)$$

325 The following scenarios may occur at t_k :

- Scenario A: $y \geq L$, corrective replacement is carried out at t_k . The degradation level after the CR is 0. The system degradation increases from 0 at t_k^+ (instant just after the CR) to x at t_{k+1} . In this case, $p(x | y, v)$ is just equal to $f_{\alpha \cdot \Delta T, \beta}(x)$;
- 330 • Scenario B: $y < L, y \geq x^0$ (equation 24), and $v \leq r^0$ (equation 25), a preventive replacement is carried out. As in the previous scenario, $p(x | y, v)$ is equal to $f_{\alpha \cdot \Delta T, \beta}(x)$;
- Scenario C: $y < L, y \geq x^0$, and $v > r^0$, the system is left as it is. The system degradation increases from y to x . $p(x | y, v)$ is therefore equal to
335 $f_{\alpha \cdot \Delta T, \beta}(x - y)$;
- Scenario D: $y < x^0$, the system is left as it is. The system degradation increases from y to x . The expression of $p(x | y, v)$ is the same as that in the scenario C: $p(x | y, v) = f_{\alpha \cdot \Delta T, \beta}(x - y)$.

From these above analysis, $p(x | y, v)$ can be calculated as follows

$$\begin{aligned}
 p(x | y, v) = & \mathbb{I}_{\{y \geq L \text{ or } (y \in [x^0, L) \text{ and } v \leq r^0)\}} \cdot f_{\alpha \cdot \Delta T, \beta}(x) \\
 & + \mathbb{I}_{\{y < x^0 \text{ or } (y \in [x^0, L) \text{ and } v > r^0)\}} \cdot f_{\alpha \cdot \Delta T, \beta}(x - y)
 \end{aligned} \tag{26}$$

where, $\mathbb{I}_{\{x\}}$ is an indicator function defined as

$$\mathbb{I}_{\{x\}} = \begin{cases} 1 & \text{if } x \text{ is true} \\ 0 & \text{if } x \text{ is false} \end{cases} \tag{27}$$

Stationary distribution. To determine the stationary law π_1 as well as π , equation 22 is rewritten as the following thanks to equations 23 and 26:

$$\begin{aligned}
\pi_1(x | u) \cdot \pi_2(u) &= f_{\alpha \cdot \Delta T, \beta}(x) \cdot \int_0^\infty \left(\int_L^\infty \pi_1(y | v) dy \right) \cdot \pi_2(v) \cdot f_{r_t | r_s}(u | v) dv \Bigg\} \text{Scenario A} \\
&+ f_{\alpha \cdot \Delta T, \beta}(x) \cdot \int_0^{r^0} \left(\int_{x^0}^L \pi_1(y | v) dy \right) \cdot \pi_2(v) \cdot f_{r_t | r_s}(u | v) dv \Bigg\} \text{Scenario B} \\
&+ \int_{r^0}^\infty \left(\int_{x^0}^{\min(L, x)} \pi_1(y | v) \cdot f_{\alpha \cdot \Delta T, \beta}(x - y) dy \right) \cdot \pi_2(v) \cdot f_{r_t | r_s}(u | v) dv \Bigg\} \text{Scenario C} \\
&+ \int_0^\infty \left(\int_0^{\min(x^0, x)} \pi_1(y | v) \cdot f_{\alpha \cdot \Delta T, \beta}(x - y) dy \right) \cdot \pi_2(v) \cdot f_{r_t | r_s}(u | v) dv \Bigg\} \text{Scenario D}
\end{aligned} \tag{28}$$

In the above equation, the expressions of $f_{\alpha \cdot \Delta T, \beta}(\cdot)$, $f_{r_t | r_s}(\cdot | \cdot)$, and π_2 are well defined by equations 1, 3, and 21. Equation 28 can be rewritten in form $\pi_1 = g(\pi_1)$. It can be therefore solved numerically based on the fixed point method. The numerical algorithm and its validation will be discussed in more detail in Appendix B.

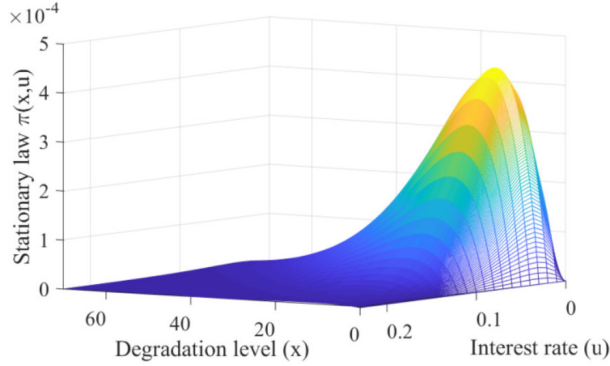


Figure 3: Illustration of the stationary law π

Figure 3 represents an example of the stationary law π when $\alpha = 0.4$; $\beta =$
 0.1; $L = 15$; $\Delta T = 5$; $R_0 = 0.7$; $P_0 = 0.85$; $a = 0.4360$; $b = 0.0612$; $c = \sqrt{a \cdot b}$;
 and $r_0 = 0.08$.

5. Maintenance performance evaluation and optimization

Thanks to the semi-regenerative property of the $(Y_t)_{t \in \mathbb{R}^+}$ process, the calculation of the long-run expected maintenance cost rate can be simplified by considering only a single semi-regenerative cycle. Equation 18 can be rewritten as follows [54]:

$$\begin{aligned} C_\infty(\Delta T, R_0, P_0) &= \lim_{t \rightarrow \infty} \frac{\mathbb{E}_0[C(t)]}{\mathbb{E}_0[\tau(t)]} = \lim_{t \rightarrow \infty} \frac{\frac{\mathbb{E}_0[C(t)]}{t}}{\frac{\mathbb{E}_0[\tau(t)]}{t}} = \frac{\frac{\mathbb{E}_\pi[C(\Delta T)]}{\Delta T}}{\frac{\mathbb{E}_\pi[\tau(\Delta T)]}{\Delta T}} = \frac{\mathbb{E}_\pi[C(\Delta T)]}{\mathbb{E}_\pi[\tau(\Delta T)]} \\ &= \frac{c_i + \mathbb{E}_\pi[C_p(\Delta T)] + \mathbb{E}_\pi[C_c(\Delta T)] + c_d \cdot (\Delta T - \mathbb{E}_\pi[\tau(\Delta T)])}{\mathbb{E}_\pi[\tau(\Delta T)]} \end{aligned} \quad (29)$$

where, $C_p(\Delta T)$, $C_c(\Delta T)$, and $\tau(\Delta T)$ are the cumulative PR cost, CR cost and operational time in a semi-regenerative cycle ΔT ; $\mathbb{E}_\pi(\cdot)$ is the expected values with respect to the stationary law π .

The expected quantities $\mathbb{E}_\pi[C_c(\Delta T)]$, $\mathbb{E}_\pi[C_p(\Delta T)]$, and $\mathbb{E}_\pi[\tau(\Delta T)]$ can be calculated thanks to the stationary law π . The expected CR cost is equal to the product of the CR cost and the CR probability at t_{k-1} .

$$\mathbb{E}_\pi[C_c(\Delta T)] = \int_L^\infty \left(\int_0^\infty (\epsilon_c \cdot e^v) \cdot \pi(y, v) dv \right) dy = \epsilon_c \cdot \int_L^\infty \left(\int_0^\infty e^v \cdot \pi(y, v) dv \right) dy \quad (30)$$

Similarity, the expected PR cost is equal to the product of the PR cost and the PR probability at t_{k-1} .

$$\mathbb{E}_\pi[C_p(\Delta T)] = \int_{x^0}^L \left(\int_0^{r^0} (\epsilon_c \cdot e^v) \cdot \pi(y, v) dv \right) dy = \epsilon_p \cdot \int_{x^0}^L \left(\int_0^{r^0} e^v \cdot \pi(y, v) dv \right) dy \quad (31)$$

To calculate the expected operational time in $[t_{k-1}, t_k]$, the two following cases are considered:

- The system is replaced at t_{k-1} (Scenarios A and B). Its degradation level is reset to zero after the replacement. The expected operational time of

the system in $[t_{k-1}, t_k)$ is

$$\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | 0) dt \quad (32)$$

- The system is left as it is at t_{k-1} (Scenarios C and D). Its degradation level remains unchanged and is equal to y . The expected operational time of the system in $[t_{k-1}, t_k)$ is

$$\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | y) dt \quad (33)$$

According to the above analysis, the expected operational time can be calculated as

$$\begin{aligned} \mathbb{E}_\pi[\tau(\Delta T)] = & \left(\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | 0) dt \right) \cdot \int_0^\infty \left(\int_L^\infty \pi(y, v) dy \right) dv \quad \left. \vphantom{\int_0^{\Delta T}} \right\} \text{Scenario A} \\ & + \left(\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | 0) dt \right) \cdot \int_0^{r^0} \left(\int_{x^0}^L \pi(y, v) dy \right) dv \quad \left. \vphantom{\int_0^{\Delta T}} \right\} \text{Scenario B} \\ & + \int_{r^0}^\infty \left(\int_{x^0}^L \left(\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | y) \cdot \pi(y, v) dt \right) dy \right) dv \quad \left. \vphantom{\int_0^{\Delta T}} \right\} \text{Scenario C} \\ & + \int_0^\infty \left(\int_0^{x^0} \left(\int_0^{\Delta T} R_{t_f|X_{t_{k-1}}}(t | y) \cdot \pi(y, v) dt \right) dy \right) dv \quad \left. \vphantom{\int_0^{\Delta T}} \right\} \text{Scenario D} \end{aligned} \quad (34)$$

Let's consider an example in which $\alpha = 0.4$; $\beta = 0.1$; $L = 15$; $\Delta T = 5$; $R_0 = 0.7$; $P_0 = 0.85$; $a = 0.4360$; $b = 0.0612$; $c = \sqrt{a \cdot b}$; $r_0 = 0.08$; $c_d =$
355 35; $c_i = 5$; $\epsilon_p = 60$; and $\epsilon_c = 75$. The long-run expected maintenance cost rate calculated by using both the Monte Carlo simulation (100 times) and the proposed analytical method (based on the semi-regenerative property). The obtained results sketched in figure 4 show that the mean of maintenance cost rate determined by the simulation converges quickly to the C_∞ calculated by
360 the analytical method.

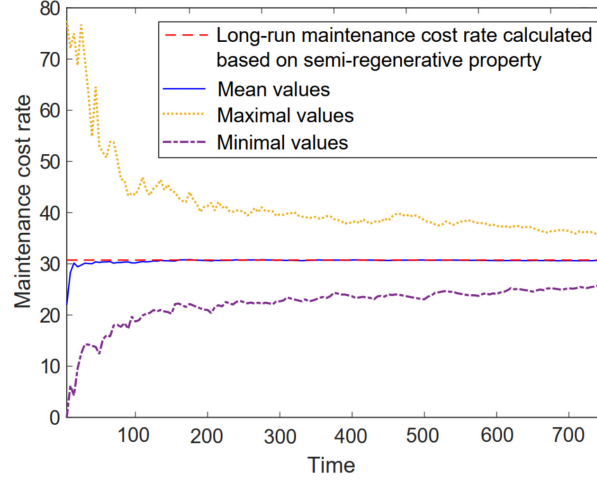


Figure 4: The convergence of C_∞ obtained by Monte Carlo simulations

Finally, the optimal parameters of the proposed policy, denoted ΔT^* , R_0^* , P_0^* , can be determined by minimizing the long-run expected maintenance cost rate.

$$(\Delta T^*, R_0^*, P_0^*) = \underset{(\Delta T, R_0, P_0)}{\operatorname{argmin}} C_\infty(\Delta T, R_0, P_0) \quad (35)$$

6. Numerical examples

In this section, we first verify the necessity of taking into account the market price volatility in PdM decisions. The performance of the proposed PdM policy is then analyzed with respect to different volatility levels and cost configurations.

365 6.1. Necessity of taking into account the market price volatility in PdM decisions

We consider herein two PdM policies:

- **The proposed PdM policy, named $(\Delta T, R_0, P_0)$ -policy.** The PdM decisions are done based on both the system conditional reliability and the interest rate level (see subsection 3.2);

- **$(\Delta T, R_0)$ -policy.** The price volatility does not considered in the decision-making because P_0 was removed. At each inspection time, the system is replaced correctively if $x \geq L$ and preventively if $R_{t_f|X_{t_k}}(t_{k+1} | x) < R_0$ and $x < L$.

The two policies were applied to the same system. The system degradation is described by the gamma process with $\alpha = 0.1$ and $\beta = 0.1$. The system is considered to be failed when its degradation level reaches the limit $L = 15$. The parameters of the maintenance cost model are given in table 1.

Interest rate parameters	$a = 8; b = 0.40; c = \sqrt{2ab}; r_0 = 0.5;$
Maintenance costs	$\epsilon_p = 60.65; \epsilon_c = 75.82; c_i = 10; c_d = 35.$

Table 1: Parameters of the maintenance cost model

Note that the above values were chosen arbitrarily to verify the effectiveness of the proposed maintenance policy.

In order to find the optimal values of the two policies' parameters, the exhaustive search method was applied. This method is an exact one in which all possible solutions in the search space are examined. In our case, the search space is defined as follow : ΔT is between 1 to 10 with a step of 1 and P_0 and R_0 are between 0 to 1 with a step of 0.1. Note that we can choose the same intervals with smaller steps but the computational time will be significantly increased. Table 2 reports the optimal configurations of the two PdM policies obtained by the exhaustive search.

Policies	Optimal parameters	C_∞
$(\Delta T, R_0)$ -policy	$\Delta T^* = 4, R_0^* = 0.7$	11.36
$(\Delta T, R_0, P_0)$ -policy	$\Delta T^* = 4, R_0^* = 0.8, P_0^* = 0.7$	10.83

Table 2: Optimal parameters of $(\Delta T, R_0)$ -policy and $(\Delta T, R_0, P_0)$ -policy

The results show that by taking into account the market price volatility, the

proposed maintenance policy helps to save around

$$\text{Saving} = \frac{(11.36 - 10.83)}{11.36} \cdot 100 \approx 5\% \quad (36)$$

390 Different shapes and iso-level curves presented in figure 5 confirm the existence of the optimal values of the policy parameters. With respect to the given data, we have $\Delta T^* = 4$, $R_0^* = 0.8$, and $P_0^* = 0.7$.

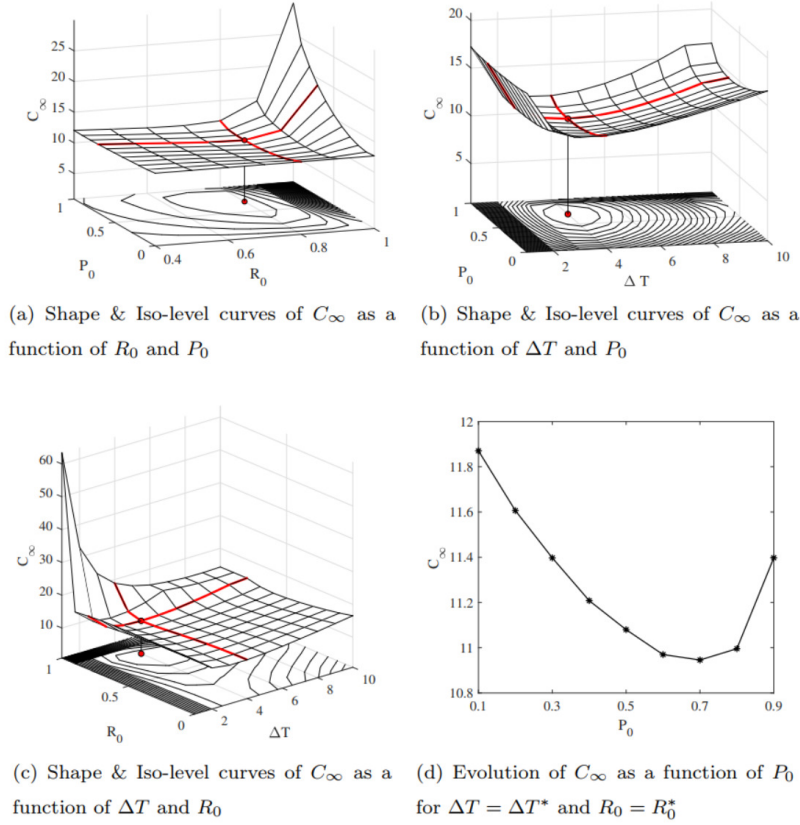


Figure 5: Shapes & Iso-level curves of C_∞ when $(\Delta T, R_0, P_0)$ -policy is applied

Figure 5d clearly shows that the choice of P_0 , control parameter associated with the market price volatility level, has strong impacts on the maintenance

395 cost.

From these above analyses, we can conclude that taking into account the market price volatility is necessary to improve the PdM decision making. The sensitivity analysis will be carried out in the next subsections to reinforce this conclusion.

400 6.2. Sensitivity analysis to the price volatility characteristics

In this subsection, the saving, defined by equation 36, is analyzed with respect to various trends of the interest rate, representing factor of the market price volatility. To this end, two policies $(\Delta T, R_0, P_0)$ -policy and $(\Delta T, R_0)$ -policy were applied to the same system with data given in table 1; however, the parameters of interest rate were varied as follow:

Sensitivity analysis to	Figures	Interest rate parameters
Mean reversion speed	figure 6a	$a = \{2, 4, 6, 8\}; b = 0.2; c = \sqrt{2ab}; r_0 = 0.3$
Volatility rate	figure 6c	$a = 8; b = 0.45; c = \{\sqrt{2ab}, \sqrt{ab}, \sqrt{ab/2}, \sqrt{ab/4}\}; r_0 = 0.5$
Interest rate level at $t = 0$	figure 6d	$a = 8; b = 0.5; c = \sqrt{2ab}; r_0 = \{0.1, 0.2, 0.3, 0.4, 0.5\}$

405

The obtained results are plotted and represented in figures 6a, 6b, 6c and 6d. Figures 6b and 6c show that the long-run parameters c and b have strong impacts on the performance of the proposed policy. The higher the levels of the volatility rate and long-run mean are, the higher the fluctuation level of the maintenance costs is, and therefore the higher the saving is. This means that the proposed policy is especially suitable for the cases in which the price volatility level is high. Otherwise, the saving does not much depend on the short-term parameters a and r_0 (see figures 6a and 6d). This conclusion seems to be reasonable since our maintenance objective (the maintenance cost rate) is evaluated for long-run horizons. The impacts of temporary phenomena driven by the short-term parameters (a and r_0) are then less important when compared to that of permanent phenomena driven by the long-term parameters (b and c).

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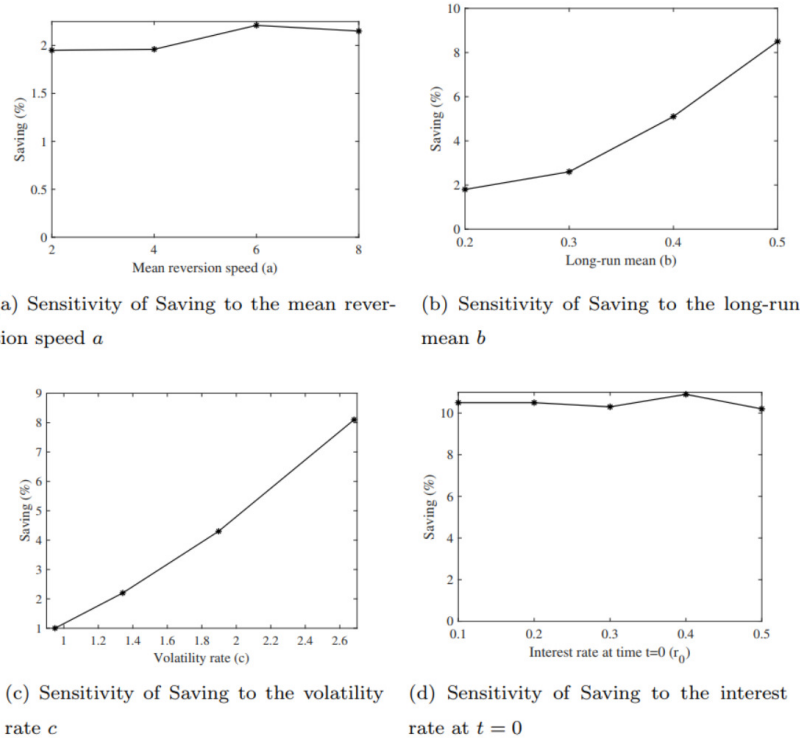


Figure 6: Sensitivity analysis of Saving to the price volatility characteristics

6.3. Sensitivity analysis to the other maintenance costs

Let now analyze the proposed policy with respect to the different values of the inspection cost, the downtime cost rate, and the ratio between the CR and CR costs. For more details, the data given in table 1 are remained unchanged, except the maintenance costs. The parameters used for the sensitivity analysis to the maintenance costs are reported in table 3.

Sensitivity analysis to	Figures	Maintenance costs
Inspection cost	figure 7	$\epsilon_p = 60.65$; $\epsilon_c = 75.82$; $c_d = 35$; $c_i = \{0, 10, \dots, 160\}$
Downtime cost rate	figure 8	$\epsilon_p = 60.65$; $\epsilon_c = 75.82$; $c_i = 10$; $c_d = \{35, 70, \dots, 595\}$
Ratio between CR and PR costs	figure 9	$\epsilon_p = 60.65$; $c_d = 35$; $c_i = 10$; $\epsilon_c/\epsilon_p = \{1, 2, \dots, 15\}$

Table 3: Parameters used in the sensitivity analysis of Saving to the maintenance costs

Figure 7a shows that the performance of the proposed policy decreases when

the inspection cost increases. These results can be explained by looking at the sensitivity analysis of the decision parameters. When the inspection cost increases, the inspection cycle ΔT^* increases to reduce the inspection frequency as well as the total inspection cost (figure 7b); however, the system failure will occur more frequently. Consequently, R_0^* decreases (figure 7c) and P_0^* increases (figure 7d) to promote the maintenance at each inspection and to reduce the system failure probability. Finally, when the inspection cost is too expensive ($c_i \geq 160$), P_0^* equals one (figure 7d), i.e. the proposed maintenance policy and $(\Delta T; R_0)$ -policy provide the same performance (figure 7a).

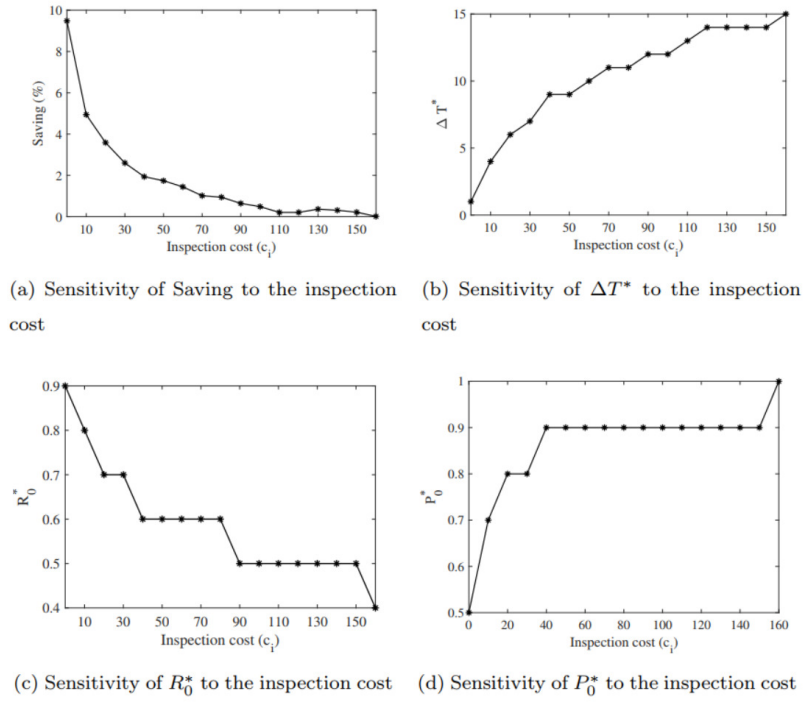


Figure 7: Sensitivity of Saving and decision parameters to the inspection cost

Figure 8 represents the results obtained from the sensitivity analysis of the proposed policy to the downtime cost rate. Figure 8a shows that the performance of the proposed policy decreases when the downtime cost rate increases.

In opposition to the case of inspection cost, the inspection cycle decreases to reduce the downtime (figure 8b). Given that the system has been already inspected frequently, R_0^* is high to avoid over-maintenance (figure 8c). Finally, P_0^* (figure 8d) increases to reduce the failure probability in the next operational period.

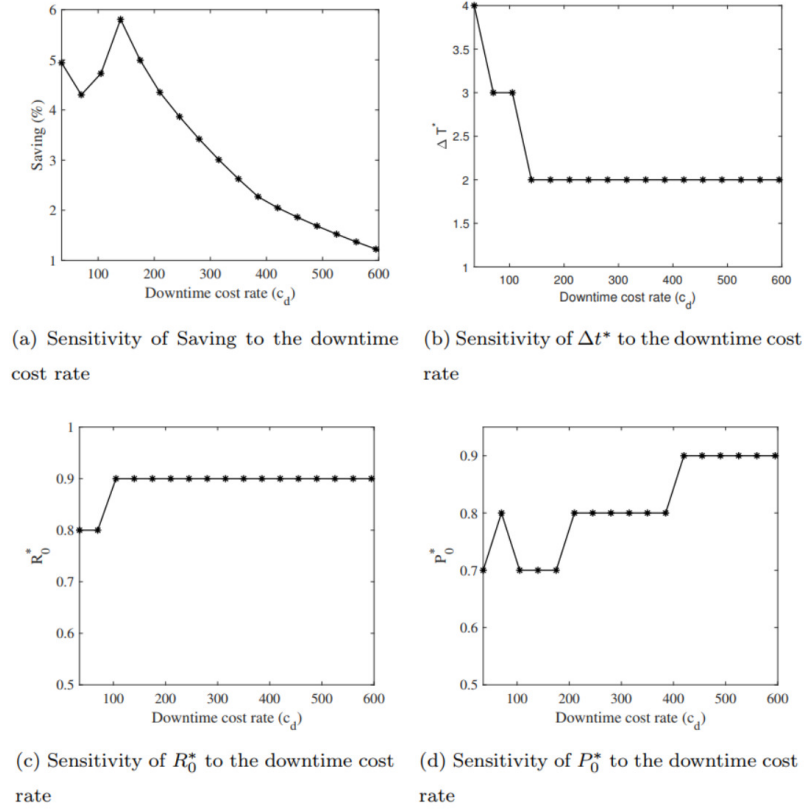


Figure 8: Sensitivity of Saving and decision parameters to the downtime cost rate

Figure 9 represents the results obtained from the sensitivity analysis of the proposed policy to the ratio between CR cost and PR cost. Figure 9a shows that the performance of our policy decreases when the ratio increases. The increase of the ratio means that the CR cost increases. However, the increase of CR cost has less impacts on the inspection cycle (figure 9b) or R_0^* (figure 9c) when

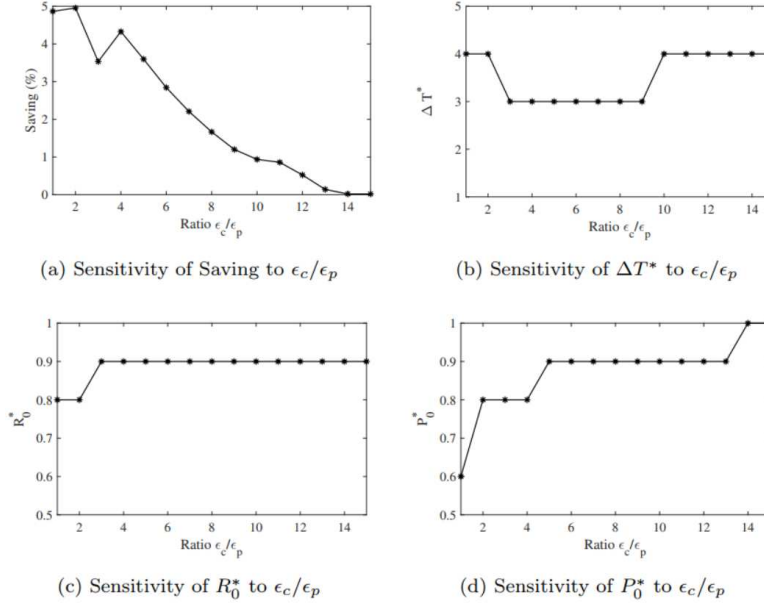


Figure 9: Sensitivity of Saving and decision parameters to the ratio between CR and PR costs

compared to that of the downtime cost rate. Otherwise, P_0^* increases clearly when the CR cost increases. This phenomenon is reasonable because the delay
of a PR at an inspection time may lead to a very expensive CR.

Finally, it should be noted that the performance of our policy is always better than that of $(\Delta T; R_0)$ -policy. It provides the same performance level as $(\Delta T; R_0)$ -policy, but only in the worst cases when the inspection cost, the downtime cost rate or the CR cost are extremely high.

7. Conclusions

In this paper, we propose a PdM policy which allows to take into consideration market price volatility. An analytical method has been developed based on the semi-regenerative property of the maintained system to estimate the performance of the proposed policy. Different numerical studies show that taking

460 into account the price volatility in PdM decision-making can help to reduce significantly the maintenance cost. In addition, the effectiveness of the proposed policy is also verified. From a practical point of view, the proposed policy is suitable for the cases where the level of volatility is high and for the systems in which maintenance costs such as inspection cost, downtime cost rate, or CR
465 cost are not extremely expensive. It is therefore applicable to most production systems. Our future research works will focus on the application of the proposed PdM model to the real systems with real data.

Appendices

Appendix A : Non-central chi-squared distribution

The p.d.f of the non-central chi-squared distribution is defined as

$$f(x, k, \lambda) = \frac{1}{2} e^{-\frac{\lambda+x}{2}} \left(\frac{x}{\lambda}\right)^{\frac{k}{4}-\frac{1}{2}} I_{\frac{k}{2}-1}(\sqrt{\lambda x}) \quad (37)$$

470 where, k is an integer number representing the number of degrees of freedom, and $\lambda > 0$ is the non-centrality parameter.

A random variable X follows the non-central chi-square distribution can be expressed as the sum of the squared of k independent, normally distributed random variables X_i with means μ_i , and unit variances [55].

$$X = \sum_{i=1}^k X_i^2 \quad (38)$$

The non-centrality parameter λ is related to the mean of the random variables X_i as

$$\lambda = \sum_{i=1}^k \mu_i^2 \quad (39)$$

Appendix B: Adapted fixed point method for solving of the invariant equation

Algorithm 1: Adapted fixed point method for solving equation 28

Step 1. Initialize $\pi_1(y | v)$ with any probability distribution function,
e.g. p.d.f of the gamma distribution:

$$f_{\alpha \cdot \Delta T, \beta}(y) = \frac{1}{\Gamma(\alpha \cdot \Delta T)} \cdot \beta^{\alpha \cdot \Delta T} \cdot y^{\alpha \cdot \Delta T - 1} \cdot e^{-\beta y} \cdot 1_{\{y > 0\}}. \quad (40)$$

Step 2. Introduce $\pi_1(y | v)$ into the right-hand side (RHS) of
equation 28. Calculate the RHS and set it as the value of $\pi_1(x | u)$.

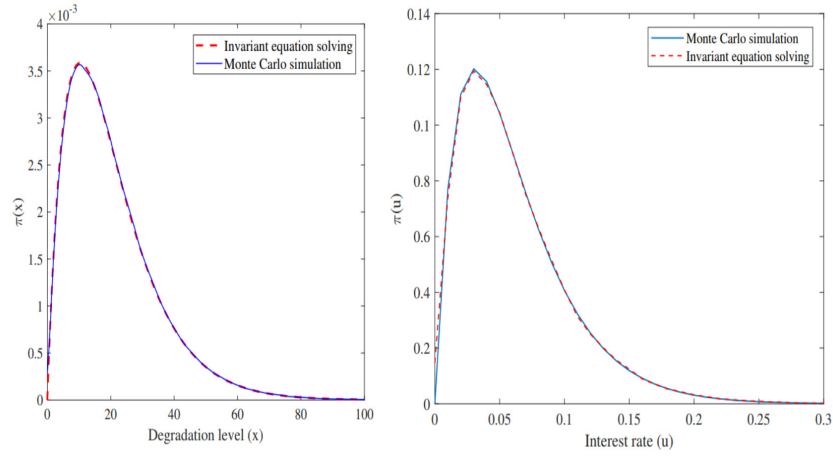
Step 3. Stop the algorithm when the maximum number of iterations
is reached or when the difference between $\pi_1(x | u)$ and $\pi_1(y | v)$ is
small enough, if not set $\pi_1(y | v) = \pi_1(x | u)$ and return to step 2.

Step 4. Calculate $\pi(x, u) = \pi_1(x | u) \cdot \pi_2(u)$.

To verify the accuracy of equation 28 as well as the above numerical method,

475 Monte Carlo simulation is used. For more detail, Algorithm 1 and the Monte
Carlo simulation were applied to estimate the stationary laws given the following
parameters: $\alpha = 0.4$; $\beta = 0.1$; $L = 15$; $\Delta T = 5$; $R_0 = 0.7$; $P_0 = 0.85$;
 $a = 0.4360$; $b = 0.0612$; $c = \sqrt{a \cdot b}$; and $r_0 = 0.08$. The obtained results shown
in figures 10a and b confirm the coherence between the analytical algorithm and
480 the simulation one.

Finally, it should be noted that if the formation of π_2 (equation 21) is not
available, the stationary laws can be determined by using Algorithm 1 to solve
directly the equation 19. However, its convergence speed is low. Indeed, w.r.t
the above parameters, the algorithm converges after 33 iterations when the
485 formulation of π_2 is available; otherwise in case when the formulation of π_2 is
unknown, the algorithm needs up to 54 iterations to reach its convergence.



(a) Stationary law of the degradation level (b) Stationary law of the interest rate

Figure 10: Stationary laws by using the simulation and by the invariant equation solving

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