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**ABSTRACT**

The Disassembly Economic Order Quantity (DEOQ) problem is to determine the quantities of a product to be disassembled at different times over an infinite planning horizon by considering ordering, operation and inventory costs. The demands for the components are independent, which can lead to accumulations of unnecessary inventories over time. This paper proposes the models which integrate price sensitive demands and disposal decisions in DEOQ problems to maximize the profit of disassembly systems without inventory accumulations. Three models are developed and analyzed to obtain solution approaches that give prices, replenishment cycle time (or, equivalently, order quantity) and disposal quantity. The inventory policy integrating both pricing and disposal decisions allows higher profits to be achieved. A numerical experiment shows its efficiency and highlights its potential implementation in practical cases.

**KEYWORDS**

EOQ Model; Disassembly system; Disassembly EOQ (DEOQ) model; Disposal; Pricing; Price-sensitive demand.

## 1. Introduction

In recent years, increasing concerns about environmental issues are forcing companies to be more responsible for their products and to recover them more efficiently. Furthermore, shorter product life cycles and shifts in consumer preferences result in higher product return flows, causing faster waste production and depletion of natural resources (El Saadany and Jaber [1] and [2]). In this situation, the regulation and leg-

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islation concerning the processing of End-Of-Life (EOL) products force companies to achieve higher recovery of their used products. This aims at making dismantling, and recycling of EOL products more environmentally friendly. For example, some directives of the European Union (EU) such as End-of-Life-Vehicles (ELVs) and Waste-Electrical and Electronic-Equipment (WEEE) will put more pressure on companies to solve the problem of waste management by recycling and re-use.

Product recovery is one of the options which aims to generate new parts or raw materials by separating and processing the EOL products. It is becoming increasingly important due to its role in saving resources and minimizing the impact of EOL products on the environment (Godichaud and Amodeo [3], Liu and Zhang [4]). Achieving both high recyclability and a positive economic balance is a challenging issue in product recovery operations, and the selection of recycling processes with specific economic and environmental benefits is very important. This encouraged some researchers to propose appropriate new models and methods to increase opportunities for cost saving and maximizing profit.

Disassembly is considered as an important step in product recovery since most returned products are disassembled prior to re-manufacturing, recycling or disposal (Kim and Xirouchakis [5]). The disassembly processes consist of generating components from the EOL products that can be allocated to a recovery channel which generates demands and revenues, or to other channels which reduce environmental impacts. However, the economic gain can be small between revenues and disassembly costs. Efficient planning of disassembly systems can increase opportunities for cost saving and make them more profitable.

Disassembly systems have special characteristics that make them challenging for planning decisions ([6]):

- The product is decomposed to meet multiple demands for parts or components,
- The demands for components are independent and not necessarily well balanced,
- The disassembly operation generates all the parts or components simultaneously.

These features imply that the quantity of product to be disassembled is not necessarily equal to the number of requested components and an unnecessary surplus inventory is likely to be generated after each disassembly operation. It can be held to satisfy future demands or be disposed of in a conscious environmental way in real industrial cases. Several solutions can be proposed to help managers to handle this surplus inventory, such as lost sales, disposal, or demand balancing by pricing.

Inventory policy can be critical within specific industries with a high return rate. The ELVs recycling sector is an example, where the EOL vehicles will be disassembled into their parts and materials, such as engines, doors, seats, tyres, plastics, metals, etc. All obtained parts and materials are checked to ensure that they are in good working order, but they are also identified, catalogued and stored for better management and traceability in order to guarantee optimal safety at the best price. According to the EU, all ELVs would need a rate of reuse and recovery of not less than 95% by no later than 1 January 2015 (the European Directive 2000/53/EC). Several costs are involved such as purchasing, disassembly operation, inventory holding, disposal and disassembly order costs. The companies must provide optimal policies so that the ELVs can be efficiently dismantled. They can sell the obtained parts on the secondary market in order to make the disassembly system profitable. This paper considers the effect of both inventory and pricing policies on the profitability of disassembly systems. The use of pricing associated with demand and surplus inventory decisions help companies manage their disassembly operations and optimize inventory policies.

The remainder of this paper is organized as follows: Section 2 presents a review of related literature. Section 3 describes the problem, followed by the model assumptions and notations, and develops the profit function with constraints for the disassembly system. In Section 4, we develop a solution approach to determine prices (or alternatively demands) for the components, the reorder interval of component inventories and order quantities. The solution approach is developed gradually by considering inventory costs, disposal decisions and upper bounds on the demands. The results of a numerical study that compares the models for the profit maximization problem (with and without a disposal option) are presented in Section 5. Section 6 provides a summary of the paper with some concluding remarks.

## 2. Related literature

This section reviews the literature related to the problem. Three research fields are identified to position our contributions: disassembly scheduling, disassembly economic order quantity (DEOQ) and EOQ models with price sensitive demand. The first two highlight inventory issues in disassembly systems, and the last one considers the opportunity of considering pricing in EOQ models. We note that there are few works on EOQ models adapted for disassembly systems, despite their advantages for real-case applications, and we point out the differences between our problem and the works in the three mentioned research fields.

### 2.1. *Disassembly scheduling*

Disassembly problems have gained increasing attention in studies concerning product recovery during the last decade. There are several classes of problems arising in disassembly systems identified in the literature (Özceylan et al. [7]). Inventory issues are mainly considered in disassembly scheduling problems, which consist of determining the overall timing and quantities of EOL products to disassemble over a planning horizon under various assumptions (on demands, returns and costs). Most of the works in this disassembly research field consider a planning decision with time varying demands over a discrete and finite planning horizon. In this context, the seminal paper of Gupta and Taleb [8] presents MRP-like (Material Requirement Planning) procedure. It determines the quantity of a product to disassemble in order to satisfy the demands for its components based on its bill-of-material. The general specificities of disassembly scheduling are highlighted in Lee et al. [9] and several extensions of the MRP-like procedure have been made to consider multiple products with part commonalities (Taleb and Gupta [10]), lot-sizing heuristics (Barba-Gutiérrez et al. [11]) and demand uncertainties (Barba-Gutiérrez and Adenso-Díaz [12]). These papers do not consider the problem of inventory accumulation mentioned in section 1. When considering various types of planning costs, mathematical programming approaches are another approach to find optimal planning. If set-up or fixed ordering cost is not considered, on-the-shelf solvers can find optimal solutions (see for instance, Lee et al.

[9], Kongar and Gupta [13], Langella [14]). Uncertainties make the problem more difficult to solve (Inderfurth et al. [15], Inderfurth and Langella [16], Kongar and Gupta [17]). Disposal decisions, which are an option to handle inventory accumulation, are considered in Kongar and Gupta [13], Langella [14], Inderfurth and Langella [16] and Kongar and Gupta [17], but only for a one period planning horizon. When considering a multi-period model with set-up cost, the inventory accumulation has to be handled period to period. The main features and properties of the problem are reviewed in Kim et al. [18], and its different variants are presented in Slama et al. [19]. Kim et al. [18] point out the inventory surplus inherent to disassembly scheduling but, few papers consider this issue. Inventory accumulation in disassembly can be indirectly reduced by allowing lost sales or external purchasing (Godichaud et al. [20], Hrouga et al. [21], Hrouga et al. [22], Ji et al. [23]), but they don't provide explicit decisions for managing the surplus inventory in each period. More recently, disposal decisions are integrated in lot-sizing models by Tafti et al. [24] and [25]. The solutions of these models show that total planning cost can be reduced and their application potentials are improved since the surplus inventory cannot be kept indefinitely in real cases.

### 2.2. *Disassembly economic order quantity (DEOQ)*

Disassembly scheduling with inventory consideration can be addressed under EOQ-like assumptions. It comes down to a lot-sizing problem but with continuous time, infinite planning horizon and constant parameters. Historically, EOQ seems to be the oldest and the most widespread lot-sizing problem, but in disassembly it came after the variant with discrete and time varying demands. The main specificity of disassembly economic order quantity (DEOQ) is presented in Godichaud et al. [26]. The authors show that if no decision is considered for handling surplus inventory, stationary policies cannot be found and the model is difficult to apply in practice. Disposal decisions are then considered in the model, and the authors propose solution approaches to find the optimal disassembly reorder interval. The work is extended in Godichaud et al. [3] for both disposal and lost sales decisions. The results show that the proposed models can determine inventory policies which allow surplus inventories to be avoided. Recently, Godichaud et al. [27] propose several DEOQ models for the problem with variant stockouts policies, such as full backorders, full-lost sales and partial backorders. In the present paper, we propose another variant to handle inventory surplus. In fact, if the demands can be varied with respect to the price, they can be adjusted to eliminate the inventory surplus, and help firms determine profit-maximizing policies.

### 2.3. *EOQ with price sensitive demand*

In many real cases, item demands are price sensitive according to a mathematical function containing market potential and price elasticity as parameters. EOQ models with price sensitive demands have been considered in several papers, but we aim to show that, due to disassembly specificity, new models have to be proposed in the case of disassembly systems. The basic model, integrating pricing and inventory decisions under EOQ assumptions for a single item, is presented in Kunreuther and Richard [28]. The model is a maximization of mean profit per unit of time function, including revenues (function of price and demand), unit purchasing cost, inventory holding and order costs. Compared to sequential optimization, i.e. a pricing decision based on revenues and unit purchasing cost first, and then an inventory decision based on holding and order costs, the joint optimization results in higher profit. The pricing part of the problem is developed in Lau and Lau [29] by considering several price-demand functions. The authors derive the optimal price for each function in the cases of single and serial stage systems. The model with pricing and inventory decision is analyzed in detail in Ray et al. [30] with two widespread demand functions: the negative power of the price (also called iso-elastic function) and a linear function. The main result, based on the first and second order derivative of the profit function, is that the optimal price has the smallest value when the first derivative of the profit function is zero, assuming there is a contiguous price range where the profit function is positive. Mathematical sensitivity analysis is performed on all the parameters of the model to give managerial insights. A similar result is presented in Abad [31] for the same demand function and the author integrates a quantity discount mechanism for the unit purchasing cost. Teksan and Geunes [32] perform the same analytical approach as in Ray et al. [30] for systems where the supply is made according to a price sensitive rate. The supply rate is an increasing function of the supply price, which is directly related to the sales price, as the supply rate must be equal to the demand rate. Recently, Adeinat and Ventura [33] discuss the problem of integrating pricing and lot-sizing decisions by considering price-dependent deterministic demand in order to maximize the profit per time unit in a serial supply chain. In disassembly systems, the prices of several items have to be considered simultaneously in the profit function with several additional constraints, and the single item models reviewed do not apply.

The single item EOQ model with pricing decision has been extended for multi-item and multi-stage problems. In multi-item problems, the item EOQ decisions are subject to linked constraints. In Cheng [34] and Chen and Min [35], they are linked according to an inventory limit constraint (investment in storage space), and the authors derived the optimal decision. Pal et al. [36] develops multi-item models with a price break level that links all the items. The first order condition gives a closed form for the optimal decision, but the second order condition can only be checked numerically on given examples. Salvietti and Smith [37] propose a solution method for the multi-item economic lot-scheduling problem (EOQ like assumption except that all the items are produced at a given rate on a single facility), with pricing decisions and capacity constraint. In multi-stage problems, one upstream stage supplies one or many downstream stages which deal with the demands. The supplier sells the item to the retailers at a wholesale price, which is a decision variable like the sales price and the order quantity. In Abad [38], the problem with one supplier and one retailer is analyzed to compare different bargaining schemes. The problem is also studied in Weng [39] by considering quantity discount on the wholesale price. These works consider, as an example, the negative power of price demand function on the retailer. The problem is extended in Viswanathan and Wang [40] by considering different discount schemes (on quantity or on volume). Bernstein and Federgruen [41] consider the problem with one supplier and several retailers, where the retailers are in competition (the demand volume of one retailer affects the demand of the others). The demand function of each retailer is a linear function of his own price, but it is also a function of other retailers' prices (if the price of one retailer increases, the demands of other retailers increase). Different decision strategies are analyzed: centralized (all the decisions are optimized together) and decentralized (each retailer optimizes his profit function). For the decentralized problem, the results are not the same when considering the prices or the demands (Bertrand or Cournot competition) as decision variables. The conditions guaranteeing a unique equilibrium (optimal price or demand) are proposed. The disassembly case differs from the ones mentioned here due to the different coordination between items. In the disassembly problem addressed in the present paper, all the component inventories are replenished simultaneously from a single source at each operation, while the demands remain independent.

The model with EOQ assumptions and pricing decision has been extended by taking into account perishable products (or subject to deterioration) and partial backorders (customers agree to wait with respect to a function of the waiting time). The model is presented assuming general assumptions for the demand function of the price in Abad [42] and Abad [43] and, in Dye et al. [44] and Papachristos and Skouri [45], for the same model with quantity discount. Linear and iso-elastic demand functions are used in examples. A different demand function for this model is proposed in Sana [46]. We note that the most used demand functions are the linear and iso-elastic functions, which are good compromise between real case representations and computational efficiency (ease of finding optimal solutions). Some researchers consider both disposal rate and pricing decision, as in El Saadany and Jaber [1] and [2]. The authors analyse a production and remanufacturing system with the return rate as a function of EOL product price and quality level. They propose new models to help firms find an optimal policy between pure manufacturing, pure remanufacturing and mixed strategy. However, in the problem we address in this paper, the demands for components are independent and can be sold to different recovery channels (material recycling, spare part markets as well as remanufacturing processes).

Based on the reviewed works in this section, we note that the pricing decision has not been investigated in disassembly systems, specifically for handling the inherent inventory surplus, and the EOQ models with price sensitive demands do not apply to disassembly. New models and solution approaches are proposed to fill the gap between EOQ models for disassembly systems and pricing decisions in EOQ models. The practical purpose is to provide decision makers clear indications concerning the profitability of a given disassembly process.

## 3. Problem and model statements

### 3.1. *Problem statement*

Disassembly systems with two-level bills of material are considered. The first level represents the leaf items obtained by one disassembly operation and the second level represents the product. A disassembly bill of material can contain spare parts, remanufacturing or requested parts as well as material fractions that can be recycled. Disassembly operations are on the product and there is no inventory for this item. The leaf items (or components) are associated with demand function, and there is an inventory for each of them. All the inventories of leaf items are replenished simultaneously by one disassembly operation on the product. The disassembly yield of each leaf item is the number of units of leaf item obtained at each disassembly operation of one unit of EOL product. Fig. 1 represents an example of a disassembly system for a product with three leaf items. The disassembly operation is on the product (item 0), and it generates all the leaf items (I, II, and III) in the quantity noted on the edge (disassembly yields).

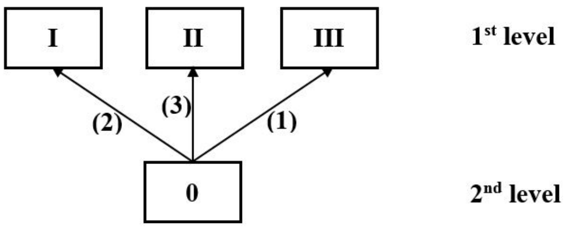


**Figure 1.** An example of a two-level disassembly structure

The following assumptions are made for all the proposed models in the paper (these are basic EOQ settings):

- demands of leaf items are independent, constant, and continuous,
- each demand is characterized by a constant rate in units per unit time (per year for example),
- the planning horizon is considered as infinite,
- the replenishment of inventories (by disassembly operations) are instantaneous,
- the disposal of a quantity of items in inventory is also instantaneous,

- the disassembly yields are known and constant,
- there is a fixed cost for each disassembly operation incurred whenever an order is placed,
- there is an inventory holding cost for each unit of an inventory per unit time.

One of the characteristics of disassembly systems is that the disassembly of one unit of EOL product generates one or more units of each of its components. It implies that, for each leaf item, the quantity of product disassembled does not necessarily give exactly the requested quantity of each component, and unnecessary stocks can be generated at each disassembly operation. By considering stationary demands, these surplus inventories accumulate with time. Disposal is an option for handling them, but it can incur additional costs. When the demands can be varied with respect to the sale price of the items (price sensitive demands), they can be optimized to maximize the profit of disassembly systems including disposal costs. A disassembly order launches disassembly operations on products and replenishes the inventories of each leaf item. The price can be defined as an increasing function of demand (or conversely the demands can be defined by using the reverse function). By integrating both disposal and pricing policies, firms can investigate the impact of price on consumer demands and examine the optimal pricing and disposal decisions.

The problem is to determine the sale prices (or alternatively the demands) of leaf items simultaneously with the disassembly policy, which sets the timing of disassembly orders and the associated quantities. The objective is to maximize the profit function. This profit function includes both the revenues generated by the sales of the leaf items and also the costs of the unit disassembly, inventory holding and order. We restrict our attention to the policies which are stationary and have the zero-inventory property. In a stationary policy, the orders are repeated according to a time cycle with a constant length. The zero-inventory property means that an inventory can be replenished only when it is zero (excluding surplus). This type of policy is commonly used in practice and studied in inventory systems (Muckstadt et al. [47]).

### 3.2. *Model statement*

The decision variables and functions used in the mathematical models are as follows:

- $Q$: Disassembly quantity per order (so that $\alpha_i Q$ units of the leaf item $i$ will be received after disassembly);
- $T$: Cycle time (time between two disassembly orders);
- $X = {}^{Q}/_{T}$: Virtual demand (used as decision variable);
- $D = \{d_1 \dots d_N\}$: Set of demands for each component;
- $\mathrm{D_i}(p_i)$, $\mathrm{P_i}(d_i)$: Demand function of price and the price function of demand for the leaf item $i$, respectively;
- $\Pi(T, X, D)$: Total mean profit per unit time;
- $\mathrm{R}(X, D)$: Total mean profit per unit time without ordering and inventory holding costs;
- $\mathrm{C}(T, D)$: Total mean ordering and inventory holding costs per unit time.

The following sets and parameters are used in this paper:

- $i = 0, 1 \dots N$: Index for the leaf items ($i = 0$ is used for the EOL product);
- $\alpha_i$: Yield of the leaf item $i$ (number of units leaf item $i$ in the EOL product);
- $h_i$: Inventory holding cost of one unit of the leaf item $i$ per unit time;
- $r_i$: Disposal cost for one unit of the leaf item $i$;
- $c$: Disassembly operations cost of one unit of the EOL product;
- $k$: Ordering cost of the EOL product;
- $d_i$: Constant demand per unit time for the leaf item $i$;
- $p_i$: Constant price for one unit of the leaf item $i$;
- $a_i$, $b_i$: Parameters of the price-demand function of the leaf item $i$.

We consider the demand of items to be deterministic with price-sensitive function. This means that the demands are defined by a decreasing function of price. In the literature, several demand functions of price are commonly studied, such as linear demand, iso-elastic, exponential and algebraic curves. Among them, linear and iso-elastic demand function of the price are the two most commonly-used mathematical functions in inventory modeling (Huang et al. [48], Ray et al. [30]). In this paper, we use an iso-elastic model, also called constant elasticity model, written as follows:

$\mathrm{D}(p) = ap^{-b} \qquad\qquad \text{where} \;\; a,\, b > 0$

This curve has a constant demand elasticity of $a$, which can be interpreted as the market potential, and $b$ is the price elasticity (Ray et al. [30], Lau and Lau [29]). We note that this function is not mathematically bounded (the demand tends to infinity when the price tends to zero). We also note that the function can be easily reversed to use demand as a decision variable instead of the price. We note $\mathrm{P}(d)$ price function of demand, written as $\mathrm{P}(d) = (a/d)^{(1/b)}$ where $a, b > 0$.

In disassembly planning, pricing decisions, when they are possible, are also advantageous to reduce surplus inventory by balancing the demands for the components. Fig. 2 represents the effect of considering both disposal decisions and price sensitive demand on a given leaf item inventory with two different policies. The quantity received for item $i$ is represented by $\alpha_i Q$. The first policy is represented by $\mathrm{D1_i}(p1_i)$ which is less than $\alpha_i Q/T$ and a quantity $R_i$ has to be disposed of. In the second policy, by increasing the demand of item $i$ to $\mathrm{D2_i}(p2_i)$ (or reducing the price), the disposed quantity is reduced but the inventory is increased if $T$ is fixed. The problem is to find the values of $T$ and $\mathrm{D_i}$ for all the items to obtain the maximum profit.

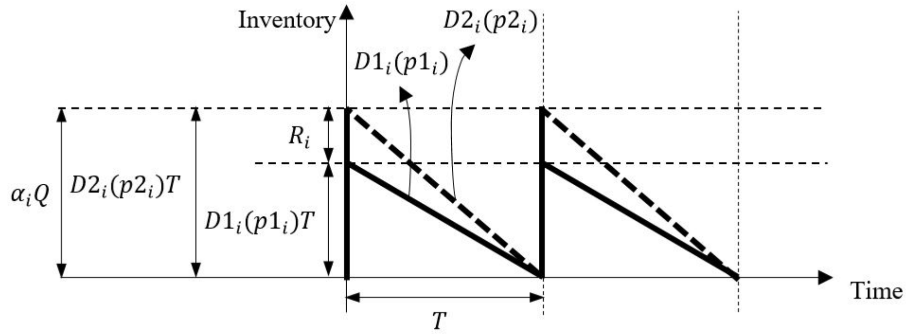


**Figure 2.** Inventory evolution of a given leaf item $i$ with respect to time by varying its demand

The problem is defined in (1) as a non-linear problem with constraints. The decision variables are $T$, $X$, and $D$ with:

- $D = \{d_1 \cdots d_N\}$ is the set of demands for each component as decision variables (the functions $\mathrm{P_i}(d_i)$ determine the associated price),
- $X = Q/T$ is the quantity of EOL product disassembled per unit time such that

all component demands can be served, which indicates virtually demands.

The objective function is the total mean profit per unit time. The constraints ensure that the disassembly quantity for each order is enough to serve all the demands.

$$Max \qquad \Pi(T,X,D) = \sum_{i=1}^{N} d_i \left(\mathrm{P_i}(d_i) + r_i\right) - X\left(c + \sum_{i=1}^{N} r_i\alpha_i\right) - \frac{k}{T} - T\left(\sum_{i=1}^{N}\frac{h_i d_i}{2}\right)$$

$$s.t. \qquad \alpha_i X \geq d_i \qquad \forall i = 1\dots N \tag{1}$$

The profit function is a difference between a profit function $\mathrm{R}(X,D)$ without ordering and inventory holding costs, and a DEOQ cost function $\mathrm{C}(T,D)$, which are studied separately below to analyze the problem:

$\Pi(T,X,D) = \mathrm{R}(X,D) - \mathrm{C}(T,D)$

with

$$\mathrm{R}(X,D) = \sum_{i=1}^{N} \mathrm{P_i}(d_i)d_i - cX - \sum_{i=1}^{N} r_i(\alpha_i X - d_i) \quad and \quad \mathrm{C}(T,D) = \frac{k}{T} + T\left(\sum_{i=1}^{N}\frac{h_i d_i}{2}\right)$$

$\mathrm{R}(X,D)$ consists of the sum of the revenues per unit of time for each leaf item, the disassembly cost per unit of time and the sum of the disposal costs per leaf item. $\alpha_i X - d_i$ is the quantity of leaf item $i$ disposed of per unit of time. $\mathrm{C}(T,D)$ is the DEOQ cost function if $D$ is fixed (Godichaud and Amodeo [3]). The maximization of $\mathrm{R}(X,D)$ is a pricing problem without inventory costs and we highlight its properties in the following section as a sub-problem of (1).

## 4. Solution approach

The Non-Linear Programming (NLP) model with constraints (equation (1)) is analyzed in this section. The solution is gradually developed with respect to the different problems, including inventory, disposal option, and demand limits considerations. The problem without inventory consideration is studied as a sub-problem. We analyze both problems with and without disposal option. The first one can be used when the disposal option is effectively not allowed, but it is also used as a sub-problem to solve the second one. Also, the problem with disposal options and limits on demands is studied.

### 4.1. *Problem without inventory costs*

By decomposing $\Pi(T, X, D)$ into the difference between $\mathrm{R}(X, D)$ and $\mathrm{C}(T, D)$, the pricing problem without inventory costs defined in (2) is studied as a sub-problem of (1). The two propositions in this section state optimal values for $X$ and $D$ and the shape of $\mathrm{R}(X, D)$ that can be used to solve the problem (1) efficiently .

$$Max \qquad \mathrm{R}(X, D) = \sum_{i=1}^{N} d_i(\mathrm{P_i}(d_i) + r_i) - X\left(c + \sum_{i=1}^{N} r_i \alpha_i\right)$$

$$s.t. \qquad \alpha_i X \geq d_i \qquad \forall i = 1 \ldots N \tag{2}$$

**Proposition 1.** *The problem (2) can be written as follows:*

$$R(X) = \left(\left(\sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X)\right) - c\right) X$$

*with only $X$ as a decision variable by setting all $d_i = \alpha_i X$ and* $\mathrm{P_i}(\alpha_i X) = (a_i/(\alpha_i X))^{(1/b_i)}$ *for the case with $b_i > 1$ for all $i = 1 \cdots N$.*

***Proof.*** See Appendix. □

Proposition 1 states that the optimal solution of (2) is to set the demands so that for each leaf item there is no surplus inventory and, therefore, no disposal. As a result, the problem has only one decision variable, which can be found efficiently according to Proposition 2.

**Proposition 2.** *For $b_i > 1$ (for all $i = 1 \cdots N$),* $\mathrm{R}(X)$ *is concave and attains its maximum at $X_0$ the unique solution of* $(\mathrm{R}'(X) = 0)$*:*

$$\sum_{i=1}^{N} \alpha_i \left(\frac{a_i}{\alpha_i X}\right)^{1/b_i} \left(\frac{b_i - 1}{b_i}\right) - c = 0$$

***Proof.*** See Appendix. □

In Proposition 2, the assumption $b_i > 1$ has been discussed in the proof of Proposition 1. Based on this analysis, a simple line search method can be used to find $X_0$.

### 4.2. *Problem without disposal decisions*

In the problem without disposal options, each demand is set as $d_i = \alpha_i X$ in order to have no surplus inventory to dispose of at the end of each replenishment cycle ($T$). The constraints in (1) are not necessary and the problem is to determine the values of $T$ and $X$ which maximize $\Pi_1(T, X)$, as defined in (3). The problem is always mathematically feasible with an iso-elastic price demand function, as the function is not bounded ($\mathrm{P_i}(\alpha_i X)$ is always defined for $X > 0$). The case with maximum potential demand is discussed later.

$$\Pi_1(T, X) = \left( \left( \sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X) \right) - c \right) X - \frac{k}{T} - (TX) \left( \sum_{i=1}^{N} \frac{h_i \alpha_i}{2} \right) \tag{3}$$

For a fixed value of $X$, the optimal reorder interval $T^*(X)$ is the minimum of the DEOQ cost function. With $T^*(X) = \sqrt{2k/(X(\sum_{i=1}^{N} h_i \alpha_i))}$, the problem is equivalent to (4):

$$\Pi_1(X) = \mathrm{R}(X) - \mathrm{C}(X) = \left( \left( \sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X) \right) - c \right) X - \sqrt{2kX \left( \sum_{i=1}^{N} h_i \alpha_i \right)} \tag{4}$$

The analysis of $\Pi_1(X)$ gives Proposition 3. It shows that a simple non-linear search method can find the optimal solution (global maximum). The proof of the proposition follows directly because it highlights the shape of the function on which the search method is based. We also note that a maximum profit, denoted by $X^*$ if it exists, is $X^* < X_0$ and the search method can be started efficiently at $X_0$ and $X^*$ is the first point such that $\Pi_1'(X) \geq 0$ by decreasing $X$ from $X_0$.

**Proposition 3.** *For $b_i > 1$, $\Pi_1(X)$ attains its maximum at the largest value of $X$ such that $\Pi_1'(X) = 0$ or at $X = 0$ if $\Pi_1(X)$ is always negative.*

***Proof.*** After analyzing the derivatives we found that there is no closed-form for the first and second derivatives.

$$\Pi_1^{'}(X) = \mathrm{R}^{'}(X) - \mathrm{C}^{'}(X) = \sum_{i=1}^{N} \alpha_i \left(\frac{b_i - 1}{b_i}\right) \mathrm{P_i}(\alpha_i X) - c - \sqrt{\frac{k(\sum_{i=1}^{N} h_i \alpha_i)}{2X}}$$

We analyze $\Pi_1(X)$ directly based on the shape of $\mathrm{R}(X)$ and $\mathrm{C}(X)$. We obtain the following results:

(1) $\mathrm{R}(X)$ is concave with maximum $X = X_0$ and $\mathrm{R}(0) = 0$.

(2) $\mathrm{C}(X)$ is strictly increasing with no stationary point.

(3) Based on (1) and (2), there are 0, 1 or 2 intersection points between $\mathrm{R}(X)$ and $\mathrm{C}(X)$ functions:

- 0 intersection point: $\Pi_1$ is always negative (which means $\mathrm{C}(X)$ always lies above $\mathrm{R}(X)$ and $\mathrm{C}(X) > \mathrm{R}(X)$) and the solution is to disassemble nothing with $X = 0$,
- 1 intersection point $X_{max}$: $\Pi_1$ is positive from $X = 0$ to $X_{max}$ and then negative (which means $\mathrm{R}(X)$ first sits above $\mathrm{C}(X)$ and then below),
- 2 intersection points $X_{min}$ and $X_{max}$: $\Pi_1$ is positive only between $X_{min}$ and $X_{max}$ (which means $\mathrm{R}(X)$ first sits below $\mathrm{C}(X)$, then above, and finally below again).

□

In real cases, upper bounds on the demands $d_i$ can be imposed according the context. A maximum demand $D_i^M$ is then defined for each item $i = 1 \dots N$. We must have $d_i \leq D_i^M$ for all $i = 1 \dots N$, and as $d_i = \alpha_i X$ for the problem without disposal, there is an additional constraint: $X \leq \underset{i}{min}\{D_i^M/\alpha_i\}$. This is an upper bound for $X$ and, if $X^* > \underset{i}{min}\{D_i^M/\alpha_i\}$, then $X = \underset{i}{min}\{D_i^M/\alpha_i\}$.

### 4.3. *Problem with disposal decisions*

Based on the results obtained in sections 4.1 and 4.2, a solution approach is proposed to solve the initial problem (1). This is a non-linear programming problem with constraints and an optimal solution satisfies the Karush-Kuhn-Tucker (KKT) conditions. The proposed search procedure uses these conditions to solve the problem by iteratively changing a solution until they are achieved. We first state the conditions and description of the search procedure.

By denoting the Lagrangian multipliers $\theta_i$ associated with each constraint $\alpha_i X \geq d_i$, the Lagrange function of the problem is defined as follow:

$$\mathrm{L}_{\Pi}(T, X, D, \theta) = \sum_{i=1}^{N} d_i(\mathrm{P_i}(d_i) + r_i) - X\left(c + \sum_{i=1}^{N} r_i\alpha_i\right) - \frac{k}{T} - \sum_{i=1}^{N} \frac{h_i d_i}{2} T \\ - \sum_{i=1}^{N} \theta_i(d_i - \alpha_i X)$$

The KKT conditions are as follows:

$$\frac{k}{T^2} - \sum_{i=1}^{N} \frac{h_i d_i}{2} = 0 \tag{5a}$$

$$\mathrm{P_i}(d_i)\left(\frac{b_i - 1}{b_i}\right) + r_i - \frac{h_i T}{2} - \theta_i = 0 \qquad \forall i = 1 \ldots N \tag{5b}$$

$$-\left(c + \sum_{i=1}^{N} \alpha_i r_i\right) + \sum_{i=1}^{N} \alpha_i \theta_i = 0 \tag{5c}$$

$$\theta_i(d_i - \alpha_i X) = 0,\ \theta_i \geq 0 \qquad \forall i = 1 \ldots N \tag{5d}$$

A first solution is found by solving the problem with $d_i = \alpha_i X$ for all $i = 1 \ldots N$. This solution is feasible for the problem (1) and easy to find, as shown in section 4.1 (only $X$ as variables without constraints). By setting $\theta_i$ with respect to (5b), the initial solution is optimal if $\theta_i \geq 0$ for all $i = 1 \ldots N$ (the other conditions (5a) to (5c) are satisfied based on proposition 3). If there are some $\theta_i < 0$, the initial solution can be improved by setting $\theta_i = 0$ and changing the related $d_i$ with respect to (5b). At this point, the following steps are repeated iteratively until the KKT conditions (5a) to (5d) are satisfied at a given threshold:

- **Step 1.** The $d_i$ with $d_i < \alpha_i X$ ($\theta_i = 0$) and $T$ are fixed. The change in $X$ is made to satisfy (5c) (the values of $\theta_i$ are set at the previous iteration).
- **Step 2.** $X$ and $T$ are fixed. With respect to (5b): update the $d_i$ with $d_i < \alpha_i X$,

compute $\theta_i$ for the $d_i$ with $d_i = \alpha_i X$ and change the $d_i$ if $\theta_i < 0$ ($\theta_i$ is changed to $\theta_i = 0$).

- **Step 3.** The $d_i$ is fixed, $T$ is set with respect to (5a).

The procedure optimizes one variable, considering the others as fixed at each step with respect to KKT conditions. It improves the profit function iteratively from the initial solution without disposal. At Step 1, only $X$ is changed (and implicitly the $d_is$ with $d_i = \alpha_i X$). If the subset of items such that $d_i = \alpha_i X$ is denoted by $S$, solving (5c) is equivalent to solving the following equation:

$$\sum_{i \in S} \alpha_i \mathrm{P_i}(\alpha_i X) \left( \frac{b_i - 1}{b_i} \right) = c + \sum_{i \notin S} \alpha_i r_i + T \sum_{i \in S} \alpha_i h_i / 2$$

The right side is constant at Step 1, and the left side is convex in $X$. Starting with the value of $X$ from the previous iteration, a new value satisfying the equality is found by simple line search. At Step 2, $d_i$ is directly found with (5b) if $\theta_i = 0$ and $T$ is fixed at the value from the previous iteration (Step 3). If $d_i = \alpha_i X$, the value of $\theta_i$ is found with (5b) and the value of $X$ found in Step 1. However, if $\theta_i < 0$, the condition (5d) is not satisfied and the related $d_i$ can be decreased to improve the profit. Once all the $d_is$ have been set in previous steps, $T$ is easily computed in Step 3 with respect to (5a).

We note that with an iso-elastic demand function, the demand is not bounded and a huge value can be optimal. This could not be practicable in real cases. The problem then is extended by imposing upper bounds for the demands which is studied in the next section.

### 4.4. *Problem with disposal decisions and limits on demands*

Without loss of integrality, we use $a_i$ as an upper bound on the demand of item $i$. This assumption is modeled with the following set of constraints added in (1):

$$d_i \leq a_i \qquad \forall i = 1 \dots N \tag{6}$$

The Lagrangian multipliers associated with each constraint (6) are denoted by $\lambda_i$.

The constraints (6) change the conditions (5b) into (5e) and add the condition (5f).

$$\mathrm{P_i}(d_i)\left(\frac{b_i-1}{b_i}\right)+r_i-\frac{h_iT}{2}-\theta_i-\lambda_i=0 \qquad \forall i=1\ldots N \tag{5e}$$

$$\lambda_i(d_i-a_i)=0,\ \lambda_i\geq 0 \qquad \forall i=1\ldots N \tag{5f}$$

The previous search method is modified by integrating these new conditions. The solution without bounds on the demands is used as an initial solution. The demands are changed to satisfy the constraints (6) if necessary. If $d_i > a_i$ in the previous solution then set $d_i = a_i$ (all other constraints are satisfied since $d_i$ is decreased and $X$ is fixed, i.e. $\alpha_i X > d_i$) and $\theta_i = 0$ since $d_i < \alpha_i X$. $T$ is iteratively updated according to the changes in $d_i s$ with (5a), (5e) and $X$ fixed.

The changes in $X$ are then made according to the condition (5c) considering $T$ and $d_i$ with $d_i < \alpha_i X$ as fixed. The changes in some $d_i s$ by setting $\theta_i = 0$ result in a change in the left side of condition (5c). If in the current solution $-\left(c+\sum_{i=1}^{N}\alpha_i r_i\right)+\sum_{i=1}^{N}\alpha_i\theta_i > 0$ (resp. $< 0$), $X$ must be increased (resp. decreased) to improve the solution. We note that after changing the initial solution to satisfy (6), if some $d_i s$ are changed to $d_i = a_i$ then $\theta_i = 0$ and the right side of (5c) becomes negative. $X$ can be decreased to improve the initial solution.

Starting from the corrected initial solution and by decreasing $X$, the search is positioned on an interval $[X_L\ X_U]$ for $X$ as follows. At first, $X_U$ is set to the solution of the problem without an upper bound on the demands and $X_L$ is set to the maximum value of $a_i/\alpha_i$ with $i$ such that $\alpha_i X > a_i$ in the current solution. Then, while condition (5c) is positive in $X_L$ and negative in $X_U$, $X_U$ is set to $X_L$ and $X_L$ is set to the next maximum value of $a_i/\alpha_i$ such that $\alpha_i X > a_i$. After each change in $X$, $D$ and $T$ are changed according to (5a) and (5e) while considering $X$ fixed. We note that, if for one item $d_i = \alpha_i X = a_i$, $\theta_i$ and $\lambda_i$ can be positive while satisfying (5e). In this case, $\theta_i$ is set to satisfy (5c) if $\theta_i + \lambda_i$ is enough with respect to (5e).

After finding the search interval $[X_L\ X_U]$ for $X$, a bisection approach on $X$ is used to find the best solution. Each value of $X$ is tested with respect to (5c) after having changed $D$ and $T$ iteratively according to (5a) and (5e) while considering $X$ fixed.

## 5. Numerical analysis

A numerical analysis is presented in this section based on several data sets. The data sets are experimental, but they can correspond to any type of real case product with several components that can be sold in different markets. For instance, disassembly centers of end-of-life vehicles have several types of glass, metal, and plastic as material outputs and headlights and engine components as spare parts outputs. These outputs correspond to the component items $i = 1$ to 10 in the data sets. The results on the experimental data sets show that the solution method is efficient enough to be applied to any real case data. The solution provided indicates if a disassembly operation is profitable, at which level and with what options. The objectives of the analysis are on four levels:

(1) The applicability of the solution method is illustrated;
(2) The effect of the disposal option and limits on the demand are highlighted;
(3) Sensitivity analysis on disposal cost and the disassembly yield are presented;
(4) Three different policies are derived to consider the case with uncertain yield.

We note from all the experiments that the proposed models and methods find the solution efficiently for all the cases with or without disposal option, and with and without limit on demands. If the disposal is allowed and the demands are limited, an optimal solution can be without disposal and all the optimal demands can be under the limit, but the solution approach finds it.

At a first level, the proposed models and methods are applied to the data presented in Table 1 to illustrate their application. It is a product with ten components. The results are presented in Table 2, with the initial solution and the last solution obtained by the procedure. Note that the disassembly quantity of EOL product per order ($Q$) can be calculated by $Q = X \cdot T$. The precision on the KKT conditions (stopping criteria) is fixed at 0.001. The computational time is negligible and the last solution is obtained after 20 iterations. The gap between the initial and last solutions is important for the tested instances, but it depends mainly on the disparity of the demand function parameters between leaf items. It is more economical to have disposal rather than more sales for items 1, 2, 4, 5 and 9 for this instance. As managerial insights, this example illustrates the data required to apply the proposed policy and show the efficiency of the solution method. Decision makers can rapidly have some indications as to the profitability of the disassembly of a product, and on the effect of changing one model parameter (a solution is easily restarted with alternative data).

**Table 1.** An illustrative instance with 10 items

| $c = 10$, $k = 9407$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 1.0 | 5.0 | 1.0 | 3.0 | 3.0 | 1.0 | 3.0 | 1.0 | 1.0 | 3.0 |
| $h_i$ | 0.92 | 0.17 | 0.85 | 0.88 | 0.41 | 0.48 | 0.26 | 0.2 | 0.56 | 0.14 |
| $a_i$ | 897.81 | 110.40 | 16592.38 | 486.58 | 1048.51 | 2541.04 | 5926.08 | 2714.32 | 1143.93 | 948.5 |
| $b_i$ | 2.3 | 1.8 | 2.6 | 2.9 | 2.6 | 2.4 | 2.1 | 1.8 | 1.5 | 3.0 |
| $r_i$ | 0.092 | 0.059 | 0.092 | 0.078 | 0.024 | 0.034 | 0.058 | 0.045 | 0.011 | 0.031 |

**Table 2.** Result for the illustrative instance

| | X | T | Q | Profit | **Solution without disposal** | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 42.1 | 7.074 | 297.815 | 207.008 | | | | | | |
| $d_i$ | 42.1 | 210.5 | 42.1 | 126.3 | 126.3 | 42.1 | 126.3 | 42.1 | 42.1 | 126.3 |
| $p_i$ | 3.78 | 1.88 | 3.78 | 2.35 | 2.35 | 3.78 | 2.35 | 3.78 | 3.78 | 2.35 |
| | X | T | Q | Profit | **Solution with disposal** | | | | | |
| | 75.062 | 7.828 | 587.585 | 579.278 | (disposal per unit time, for an item $i$: $\alpha_i X - d_i$) | | | | | |
| $d_i$ | 13.474 | 63.124 | 75.062 | 4.226 | 90.25 | 75.062 | 225.186 | 75.062 | 68.368 | 225.186 |
| $p_i$ | 6.207 | 3.172 | 2.942 | 10.277 | 2.715 | 2.942 | 1.825 | 2.942 | 3.064 | 1.825 |

Fig. 3 illustrates the shape of $\Pi_1(X)$, R(X) and C(X) for this instance and brings additional justifications of the solution approach. We note that C(X) passes above R(X) before it reaches its maximum. As shown in proposition 3, $\Pi_1(X)$ is concave while R(X) is above C(X). We also note that the solver Excel (which used a generalized reduced gradient method) finds the solution for the problem without disposal (only $X$ as decision variable), but not for the problem with disposal (with $X$, $T$ and $D$ as decision variables) starting with the initial solution. Furthermore, if the last solution is loaded in the solver, it does not improve it.

At a second level, we note in Table 2 that the disposal option is profitable in this instance. Another effect of allowing disposal is that more products are disassembled. It can be considered as negative or positive depending on whether recovery channels with proper environmental conditions are considered for the disposed items (however this is not considered in the model). Table 3 illustrates the improvement of the profit that can be achieved over the other nine examples by considering disposal option (data sets are provided in the appendix). We can note that the solution without disposal can be

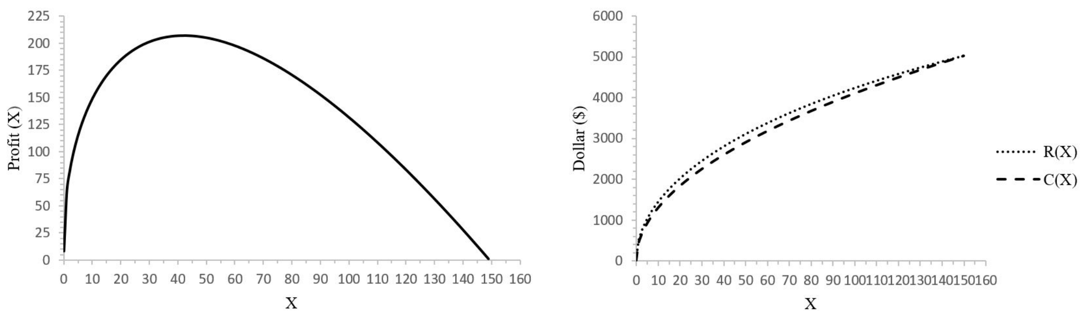


**Figure 3.** Shape of the curves for $\Pi_1(X)$, $\mathrm{R}(X)$ and $\mathrm{C}(X)$

optimal, as in example 8. It is detected in the procedure after having found the initial solution by testing the KKT conditions. For all the instances, as previously, the solver Excel finds the solution for the problem without disposal but not for the problem with disposal. The solution procedure starts with the solution without disposal option and then tries to improve it by considering disposal. We note that the solution determines whether disposal is profitable and what quantity of each item to dispose of.

**Table 3.** Illustration of the gap between initial and last solutions (without and with disposal)

| Inst. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Initial | 11653.403 | 147.504 | 2694.759 | 1224.661 | 2925.528 | 549.553 | 478.126 | 2364.457 | 3141.517 |
| Opt. | 11656.177 | 596.561 | 2722.754 | 1370.793 | 2941.938 | 620.709 | 1232.796 | 2364.457 | 3141.779 |
| Gap | 0.024% | 304% | 1.039% | 11.93% | 0.561% | 12.9%. | 157.8% | 0% | 0.008% |

As mentioned in previous sections, the limits on the demands are an additional consideration for using power iso-elastic function. In real cases, the markets have given potentials that cannot be exceeded. By considering $a_i$ as the maximum demand for each component in all examples, the solutions of examples 3, 8 and 9 are not feasible at this step of the procedure, as presented in Table 4. The procedure can be continued, as presented in section 4.4, to improve the solution within the constraints. The results are presented in Table 4. As previously, the computational time is negligible and all the KKT conditions are satisfied (with a given threshold) for the last solution found. The negative feature of power iso-elastic function (no limitation on demands) is then overcome with the proposed procedure.

At a third level, we analyze the sensitivity of the model with respect to disassembly yield and disposal cost. One of the characteristics of disassembly systems is the yield attached to each component, and we give insight into the effect of the estimation variation of this parameter on decisions and profit. Table 5 reports the results of a sensitivity analysis with respect to disassembly yield for the instance 0 (data on Table 1 in the paper). The experiment is based on the variation of each yield by one percentage point between ±25% of the initial value in 0.05% steps. The effect on the decisions ($X$, $T$ and $D$) and the profit are evaluated on average in percentage variation from the initial values (for instance, the first line indicates that a gap of -25% on one yield parameters leads to variation of -3.5% of the profit).

**Table 4.** Solution for the problem with upper bound on demands (Instances 3, 8 & 9)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Inst. 3** | | | | | | | | | | |
| | X | T | Q | Profit | **Solution without limit on demands** | | | | | |
| | 224.052 | 2.022 | 453.033 | 2722.754 | | | | | | |
| $d_i$ | 448.104 | 224.052 | 448.104 | 224.052 | 448.104 | 458.860 | 448.104 | 442.347 | 672.156 | 672.156 |
| $p_i$ | 2.723 | 8.350 | 4.409 | 2.453 | 1.129 | 2.258 | 2.569 | 1.504 | 1.966 | 0.726 |
| $a_i$ | 4058.3 | 8263.83 | 4147.93 | 3023.47 | 614.31 | 1832.65 | 2956.57 | 922.46 | 3898.07 | 311.85 |
| | X | T | Q | Profit | **Solution with limit on demands** | | | | | |
| | 207.95 | 2.17 | 451.251 | 2694.596 | **(the demand of an item must be $d_i \leq a_i$)** | | | | | |
| $d_i$ | 415.9 | 207.95 | 415.9 | 207.95 | 415.9 | 403.37 | 415.9 | 387.47 | 623.85 | 311.85 |
| $p_i$ | 2.816 | 8.724 | 4.633 | 2.517 | 1.162 | 2.436 | 2.666 | 1.619 | 2.023 | 1 |
| **Inst. 8** | | | | | | | | | | |
| | X | T | Q | Profit | **Solution without limit on demands** | | | | | |
| | 469.64 | 1.324 | 621.803 | 2364.457 | | | | | | |
| $d_i$ | 1878.56 | 939.28 | 1408.92 | 1878.56 | 939.28 | 469.64 | 469.64 | 1878.56 | 939.28 | 1408.92 |
| $p_i$ | 0.885 | 2.101 | 1.268 | 1.036 | 0.957 | 1.966 | 6.423 | 0.861 | 0.938 | 1.003 |
| $a_i$ | 1336.24 | 7514.05 | 2548.69 | 2066.2 | 838.94 | 3333.54 | 13355.88 | 1413.39 | 832.28 | 1419.53 |
| | X | T | Q | Profit | **Solution with limit on demands** | | | | | |
| | 334.06 | 1.628 | 543.849 | 2293.275 | **(the demand of an item must be $d_i \leq a_i$)** | | | | | |
| $d_i$ | 1336.24 | 668.12 | 1002.18 | 1199.15 | 668.12 | 334.06 | 334.06 | 1336.24 | 668.12 | 793.338 |
| $p_i$ | 1 | 2.373 | 1.453 | 1.223 | 1.092 | 2.211 | 7.761 | 1.030 | 1.123 | 1.231 |
| **Inst. 9** | | | | | | | | | | |
| | X | T | Q | Profit | **Solution without limit on demands** | | | | | |
| | 367.68 | 1.412 | 519.164 | 3141.779 | | | | | | |
| $d_i$ | 735.36 | 1470.72 | 367.68 | 1470.72 | 1470.72 | 735.36 | 735.36 | 367.68 | 1398.89 | 367.68 |
| $p_i$ | 2.175 | 0.518 | 3.262 | 1.250 | 1.374 | 2.193 | 2.040 | 5.893 | 0.708 | 2.538 |
| $a_i$ | 2978.69 | 548.3 | 6278.22 | 2198.47 | 3812.5 | 5238.36 | 4070.88 | 10692.41 | 726.1 | 1965.56 |
| | X | T | Q | Profit | **Solution with limit on demands** | | | | | |
| | 315.35 | 1.609 | 507.398 | 3037.924 | **(the demand of an item must be $d_i \leq a_i$)** | | | | | |
| $d_i$ | 630.7 | 548.3 | 315.35 | 1261.4 | 1261.4 | 630.7 | 630.7 | 315.35 | 726.1 | 315.35 |
| $p_i$ | 2.369 | 1 | 3.478 | 1.362 | 1.446 | 2.332 | 2.175 | 6.389 | 1 | 2.764 |

From Table 5, we can assume that if one yield increases, the reorder interval $T$ is decreased, the quantity of product disassembled per unit of time $X$ is increased and the item demands are increased. We note that the variation is contained within ±4%

for $X$, $T$ and in ±6% for the demand of items whose yield is not varied, while the variation is significantly more important for the demand of the item whose yield is varied. However, this variation has little effect on the profit.

One specificity of the proposed model is the disposal option and we conducted a sensitivity analysis with respect to disposal costs in instance 0. As for the analysis of the disassembly yield, each initial value of $r_i$ is varied by one percentage point between ±25%. The results are reported in Table 6. We note that small variations of the disposal cost have a limited effect on the decision. It has no effect for the component with no disposal options included in the initial decision (the variation of ±25% does not generate disposal for these components). In a second experiment, we vary each disposal cost more widely until it becomes unprofitable to dispose of the related component.

**Table 5.** Effect of the variation of disassembly yield on optimal decisions

| One yield variation (%) | Average profit variation (%) | Average X-value variation (%) | Average T-value variation (%) | Average $d_i$-value variation (*) (%) | Average $d_i$-value variation (**) (%) |
|---|---|---|---|---|---|
| -25 | -3.5 | -4.05 | 3.87 | -17.06 | -5.99 |
| -20 | -2.75 | -3.21 | 3.00 | -13.61 | -4.78 |
| -15 | -2.03 | -2.39 | 2.18 | -10.08 | -3.58 |
| -10 | -1.33 | -1.58 | 1.38 | -6.46 | -2.35 |
| -5 | -0.66 | -0.77 | 0.66 | -3.19 | -1.16 |
| 5 | 0.64 | 0.79 | -0.66 | 3.32 | 1.22 |
| 10 | 1.26 | 1.58 | -1.29 | 6.73 | 2.46 |
| 15 | 1.87 | 2.33 | -1.88 | 10.16 | 3.66 |
| 20 | 2.47 | 3.09 | -2.46 | 13.71 | 4.91 |
| 25 | 3.05 | 3.81 | -3.00 | 17.27 | 6.13 |

(*) for the item whose yield is varied, (**) for the item whose yield is not varied

In this second experiment, each $r_i$ is varied from 0 up to the value at which there is no more disposal for $i$. Fig. 4 presents the variation of the profit with respect to disposal cost for each item that has disposal quantity in the initial solution (items 1, 2, 4, 5 and 9). The values at which there is no more disposal for each $i$ are highlighted in Fig. 4. We note that, for each item, these values exceed the inventory holding cost ($h_i$ is a cost per unit of item per unit of time while $r_i$ is cost per unit of item). We also note that the profit decreases with respect to the increase in disposal cost, and the variation of the profit is not the same for each item: it is more important for item

4 than for item 9. Finally, an important managerial insight is that the optimization finds solutions efficiently whatever the data. Decision makers can use it to have an idea of the profitability of the disassembly process of any EOL product.

At a fourth level, we analyse the application of the proposed model with uncertain disassembly yields in practice. We compare three policies that can be applied in situations with uncertain disassembly yields. The policies are based on the model proposed in the paper. The comparison is made according to a simulation study (i.e. simulation of uncertain yields) over a planning horizon with 1000 orders (or cycles).

For the three policies, we assume that the yields are estimated and the values used for the decision optimization are expected values. We assume that the components that cannot be disassembled (or recycled) are detected at the receipt of the order. The quantity received in each component inventory can be above or below the nominal quantity (the repartition over the nominal value can be represented by a probability distribution). We also note that the three policies are stationary to maintain EOQ-like conditions: each cycle is independent and all the inventories are zero at the beginning of a cycle. We apply the model without limits on maximum demand rates (the results can be easily extended).

**Table 6.** Effect of the variation of disposal cost on optimal decisions

| One dispo. cost variation (%) | Average profit variation (%) | Average X-value variation (%) | Average T-value variation (%) | Average $d_i$-value variation (*) (%) | Average $d_i$-value variation (**) (%) |
|---|---|---|---|---|---|
| -25 | 0.19 | 0.39 | -0.17 | -0.44 | 0.39 |
| -20 | 0.16 | 0.31 | -0.14 | -0.35 | 0.32 |
| -15 | 0.12 | 0.22 | -0.10 | -0.28 | 0.22 |
| -10 | 0.08 | 0.15 | -0.06 | -0.19 | 0.15 |
| -5 | 0.04 | 0.07 | -0.03 | -0.10 | 0.07 |
| 5 | -0.04 | -0.07 | 0.03 | 0.10 | -0.07 |
| 10 | -0.08 | -0.15 | 0.06 | 0.19 | -0.15 |
| 15 | -0.11 | -0.22 | 0.10 | 0.29 | -0.22 |
| 20 | -0.15 | -0.29 | 0.13 | 0.39 | -0.29 |
| 25 | -0.19 | -0.36 | 0.16 | 0.49 | -0.36 |

(*) for the item whose disposal cost is varied, (**) for the item whose disposal cost is not varied

Based on these assumptions, the yields are simulated at each order with respect to a probability distribution. We use the data of the previous instance where $\alpha_i$ is used as the mean of uniform distribution with a range of 0.1 around $\alpha_i$ for the support of the distribution (i.e. $\hat{\alpha}_i = U(0.95\alpha_i, 1.05\alpha_i)$). $\hat{\alpha}_i$ is the observed (simulated) value of the yield of component $i$ at the beginning of a cycle, after having received the order.

**Policy P1**. The decisions $Q$, $T$ and $D$ are fixed for all orders. The quantity received in each inventory is $\hat{\alpha}_i Q$. If $d_i T > \hat{\alpha}_i Q$ then $\hat{\alpha}_i Q$ items $i$ are sold during the cycle and $d_i T - \hat{\alpha}_i Q$ sales are lost. If $d_i T \leq \hat{\alpha}_i Q$ then $\hat{\alpha}_i Q - d_i T$ items $i$ are disposed of and $d_i T$ are sold. The sale revenue, the disposal cost and the average inventory cost for each item $i$ during the cycle are then, respectively: $p_i \min\{d_i T, \hat{\alpha}_i Q\}$, $r_i \max\{0, \hat{\alpha}_i Q - d_i T\}$ and $(h_i/2)\min\{d_i T^2, (\hat{\alpha}_i Q)^2/d_i\}$.

In this policy, stockouts are allowed and are considered as lost sales without additional costs. The only effect is that fewer items can be sold during one cycle. The two other policies avoid lost sales by changing more variables at each cycle.

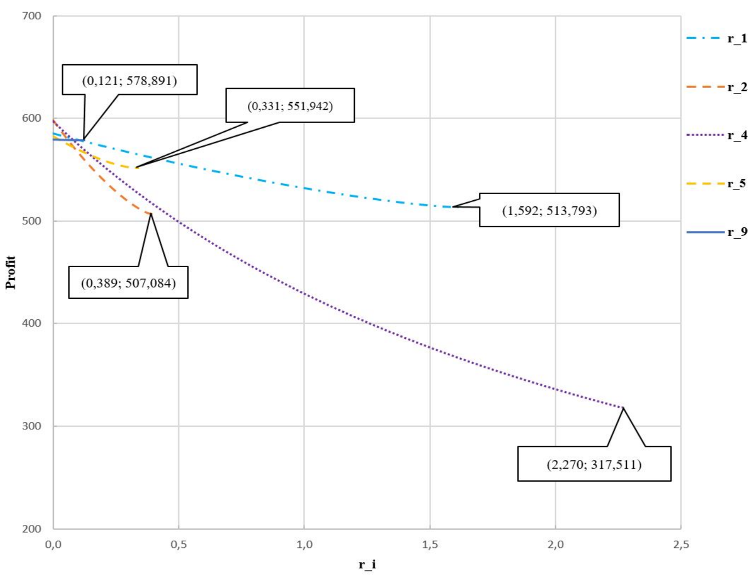


**Figure 4.** Variation of the profit with respect to the disposal cost (from 0 to the value at which there is no more disposal for item $i$)

**Policy P2**. The decisions $Q$ and $D$ are fixed for all orders but $T$ is changed every cycle to prevent lost sales. After having observed the value of the yields, the next order will arrive after $T = \min_i\{\hat{\alpha}_i Q/d_i\}$ units of time, with $T^*$ the reorder interval obtained by optimization. It means that $T$ is adjusted with respect to the first inventory that reaches zero during the cycle. The surplus inventories for other items are disposed of. The sales revenue, the disposal cost and the average inventory cost for each leaf item

$i$ during the cycle are then, respectively, $p_i d_i T, r_i(\hat{\alpha}_i Q - d_i T)$ and $(h_i/2) d_i T^2$.

In this policy, stockouts are avoided in contrast to policy P1, but variation of the reorder interval must be allowed. We also note that the variation of the cycle times, while keeping the order quantity fixed, is the same principle as for the policy $(s, Q)$ applied to the single item inventory model with uncertain demand and non-zero lead time.

**Policy P3**. The decisions $Q$ and $T$ are fixed for all orders but the demand-price decisions $D$ and the disposal quantity are optimized at each cycle, after having observed the value of the yields, with respect to equation (1) considering $T$ and $X$ fixed (NB $X = Q/T$). The first derivative of (1) with respect to $d_i$ gives the stationary point:

$$P_i(d_i) = p_i = \left(\frac{h_i T}{2} - r_i\right)\left(\frac{b_i}{b_i - 1}\right) \text{and } d_i = \max\{p_i,\ \alpha_i X\}$$

**Table 7.** Simulation study under yield uncertainty

| | Profit (per unit time) | | | | Gap (%) | | |
|---|---|---|---|---|---|---|---|
| Inst. | Static | P1 | P2 | P3 | gap_P1 | gap_P2 | gap_P3 |
| 0 | 579.28 | 562.39 | 549.30 | 578.93 | 2.9146 | 5.1749 | 0.0593 |
| 1 | 11656.18 | 11392.97 | 11223.36 | 11655.22 | 2.2581 | 3.7132 | 0.0082 |
| 2 | 596.56 | 574.61 | 553.92 | 595.87 | 3.6792 | 7.1468 | 0.1148 |
| 3 | 2722.75 | 2657.19 | 2618.95 | 2721.70 | 2.4080 | 3.8124 | 0.0385 |
| 4 | 1370.79 | 1324.79 | 1280.18 | 1370.42 | 3.3558 | 6.6103 | 0.0269 |
| 5 | 2941.94 | 2869.33 | 2818.59 | 2941.42 | 2.4680 | 4.1928 | 0.0177 |
| 6 | 620.71 | 597.31 | 567.08 | 620.27 | 3.7699 | 8.6403 | 0.0706 |
| 7 | 1232.80 | 1198.15 | 1179.01 | 1232.59 | 2.8100 | 4.3630 | 0.0169 |
| 8 | 2364.46 | 2268.11 | 2138.96 | 2365.10 | 4.0749 | 9.5371 | -0.0272 |
| 9 | 3141.78 | 3045.93 | 2963.29 | 3140.87 | 3.0509 | 5.6810 | 0.0289 |
| | | | | Min | 2.2581 | 3.7132 | -0.0272 |
| | | | | Avg. | 3.0789 | 5.8872 | 0.0355 |
| | | | | Max | 4.0749 | 9.5371 | 0.1148 |

The profit for each cycle is computed according to (1) with the modified value of $d_i$. This policy prevents stockouts as in policy P2, but it necessitates accepting price variation at each cycle.

The results of the simulation are presented in Table 7. We compared the policies according to the average profit per unit of time. The Table presents the profit found by the solution approach for all the instances studied in this paper. The gaps are calculated with respect to the profit found by the solution approach proposed in the paper (static profit). We note that, based on the assumption of the simulation study, policy P3 is always better than policies P1 & P2. It is also not surprising that policy P3 can slightly improve the profit: average yield is used for optimization and more items can be received in the simulation. This would not be the case if a maximal value were used for simulation. The drawback of P3 is that it requires a modification of the prices at each cycle, which is not always possible, and which can have a negative effect not taken into account in the simulation. Policy P1 is better than policy P2 on all instances, but it requires the possibility of stockouts. We finally note that the proposed model, initially dedicated to a deterministic context, is sufficiently efficient to be adapted to situations with uncertain quantity yields.

## 6. Summary and Conclusion

This paper provides a disassembly EOQ model that permits pricing to be used on the demands, with consideration of disposal decisions in order to handle unnecessary accumulations of components in a recovery system optimally. This surplus inventory can lead to inappropriate inventory decisions that are not environmentally sound, but disposal decisions can be applied to handle it. However, when it is possible, it can be both economically and environmentally advantageous to vary the demands if they are price sensitive according to a profit function.

A model has been developed in this paper to set the prices of components in disassembly systems in order to optimize a profit function with different cost structures. When considering only the disassembly cost or the disassembly with inventory costs (order and holding cost under EOQ assumptions), the optimality conditions have been derived showing that simple non-linear search methods can be used. Disposal decisions and costs have been considered subsequently, which has led to a non-linear problem with constraints. The KKT conditions have been derived to propose a solution approach. It shows that keeping the disposal option can lead to higher profit margins if the demands are price sensitive, depending on the data instances. The numerical examples show that the procedure provides solutions in short computational times.

In practical applications, estimation of the required data for the proposed model is a real challenge. However, based on the numerical analysis, the model seems to be robust enough to be used, in a first attempt to manage the process, with rough data. The quality or ratio of obtained parts and materials may be different after the disassembly operation. The solution obtained from the model can, however, be used in an uncertain context as shown in the numerical experiment. Further research can extend the results in several ways. In disassembly centers, the return of products can be limited and a price sensitive return function can be added to the model. Shortages can also be taken into account with different strategies and associated costs. Additionally, our model considered EOQ-like assumptions, but it could be interesting to study the effect of pricing decisions with other inventory assumptions such as dynamic lot-sizing problems, or stochastic models.

## Acknowledgments

The authors thank the departmental council of Aube (CD10), along with the European Union (FEDER) for supporting the research. They also thank the reviewers for their useful comments and suggestions.

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## Appendix

### Proof of proposition 1.

If $X$ is fixed, the problem is to maximize $\mathrm{V}(D) = \sum_{i=1}^{N} \mathrm{V_i}(d_i)$ with $\mathrm{V_i}(d_i) = (\mathrm{P_i}(d_i) + r_i)d_i$ subject to $\alpha_i X \geq d_i$ (for all $i = 1 \cdots N$). The function $\mathrm{V}(D)$ can be decomposed into $N$ independent sub-functions of $\mathrm{V_i}(d_i)$ to set the value of the $d_i$s. Based on its first derivative with respect to $d_i$, denoted by $\mathrm{V_i}^{'}(d_i)$, a stationary point of $\mathrm{V_i}(d_i)$ must satisfy $\mathrm{P_i}(d_i) + r_i + \mathrm{P_i}^{'}(d_i)d_i = \mathrm{P_i}(d_i)((b_i - 1)/b_i) + r_i = 0$. In the case of $b_i < 1$, the function is convex (the second derivative is $\mathrm{V_i}^{''}(d_i) = \mathrm{P_i}^{'}(d_i)((b_i - 1)/b_i) > 0$) and, since we must have $\alpha_i X \geq d_i$, $d_i$ must be set as small as possible, which is an uninteresting scenario. In the following we consider that $b_i > 1$ for all $i = 1 \cdots N$. In this case, the equation $\mathrm{V_i}^{'}(d_i) = 0$ has no solution and $\mathrm{V_i}(d_i)$ increases. Each $d_i$ is then chosen as large as possible, subject to the constraints $\alpha_i X \geq d_i$ which lead to $d_i = \alpha_i X$.

### Proof of proposition 2.

For $\mathrm{P_i}(\alpha_i X) = (a_i/(\alpha_i X))^{(1/b_i)}$, the first derivative is $\mathrm{P_i}^{'}(\alpha_i X) = -\mathrm{P_i}(\alpha_i X)/(b_i X)$. The first derivative of $\mathrm{R}(X)$ is then:

$$\begin{aligned}\mathrm{R}^{'}(X) &= \sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X) - c + \left(\sum_{i=1}^{N} \alpha_i \mathrm{P_i}^{'}(\alpha_i X)\right) X \\ &= \sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X) \left(\frac{b_i - 1}{b_i}\right) - c \qquad \text{(A.1)}\end{aligned}$$

For $b_i > 1$ (for all $i = 1 \cdots N$), the function $\alpha_i \mathrm{P_i}(\alpha_i X)(b_i - 1)/b_i$ then strictly decreases from $+\infty$ to 0 as $X$ varies from 0 to $+\infty$. The sum function $\sum_{i=1}^{N} \alpha_i \mathrm{P_i}(\alpha_i X)(b_i - 1)/b_i$ is then strictly decreasing from $+\infty$ to 0 and $\mathrm{R}^{'}(X) = 0$ has one solution. We obtain second derivative of function R($X$) as follows:

$$\mathrm{R}^{''}(X) = \sum_{i=1}^{N} \left( \frac{-\alpha_i \mathrm{P_i}(\alpha_i X)}{b_i X} \right) \left( \frac{b_i - 1}{b_i} \right) < 0 \qquad \text{(A.2)}$$

The second derivative of R($X$) is strictly negative so R($X$) is concave.

**Example data sets (Example 1 to 9).**

| **Inst. 1** ($c$ = 10, $k$ = 9407) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 1.0 | 5.0 | 1.0 | 3.0 | 3.0 | 1.0 | 3.0 | 1.0 | 1.0 | 3.0 |
| $h_i$ | 0.92 | 0.17 | 0.85 | 0.88 | 0.41 | 0.48 | 0.26 | 0.20 | 0.56 | 0.14 |
| $a_i$ | 897.81 | 110.40 | 16592.23 | 486.58 | 1048.51 | 2541.04 | 5926.08 | 2714.32 | 1143.93 | 948.50 |
| $b_i$ | 2.3 | 1.8 | 2.6 | 2.9 | 2.6 | 2.4 | 2.1 | 1.8 | 1.5 | 3.0 |
| $r_i$ | 0.092 | 0.059 | 0.092 | 0.078 | 0.024 | 0.034 | 0.058 | 0.045 | 0.011 | 0.031 |

| **Inst. 2** ($c$ = 10, $k$ = 7753) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 3.0 | 3.0 | 3.0 | 3.0 | 4.0 | 3.0 | 3.0 | 1.0 | 2.0 | 3.0 |
| $h_i$ | 0.48 | 0.78 | 0.16 | 0.97 | 0.85 | 0.87 | 0.34 | 0.18 | 0.21 | 0.21 |
| $a_i$ | 3983.73 | 182.9 | 1020.66 | 2660.12 | 704.53 | 1415.28 | 2230.57 | 4287.82 | 530.48 | 2996.21 |
| $b_i$ | 2.9 | 1.9 | 2.0 | 1.9 | 3.0 | 2.6 | 2.4 | 2.2 | 2.0 | 2.0 |
| $r_i$ | 0.027 | 0.016 | 0.019 | 0.064 | 0.054 | 0.089 | 0.052 | 0.099 | 0.064 | 0.057 |

| **Inst. 3** ($c$ = 10, $k$ = 5863) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 2.0 | 1.0 | 2.0 | 1.0 | 2.0 | 3.0 | 2.0 | 3.0 | 3.0 | 3.0 |
| $h_i$ | 0.98 | 0.67 | 0.87 | 0.75 | 0.4 | 0.99 | 0.44 | 0.69 | 0.44 | 0.43 |
| $a_i$ | 4058.3 | 8263.83 | 4147.93 | 3023.47 | 614.31 | 1832.65 | 2956.57 | 922.46 | 3898.07 | 311.85 |
| $b_i$ | 2.2 | 1.7 | 1.5 | 2.9 | 2.6 | 1.7 | 2.0 | 1.8 | 2.6 | 2.4 |
| $r_i$ | 0.085 | 0.069 | 0.022 | 0.051 | 0.056 | 0.071 | 0.027 | 0.029 | 0.055 | 0.041 |

| **Inst. 4** ($c$ = 10, $k$ = 9517) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 1.0 | 1.0 | 2.0 | 2.0 | 1.0 | 1.0 | 4.0 | 2.0 | 3.0 | 1.0 |
| $h_i$ | 0.98 | 0.67 | 0.87 | 0.75 | 0.4 | 0.99 | 0.44 | 0.69 | 0.44 | 0.43 |
| $a_i$ | 1922.3 | 1008.89 | 6770.25 | 569.75 | 8160.06 | 3820.94 | 2014.76 | 4528.52 | 1154.33 | 1416.09 |
| $b_i$ | 2.0 | 2.3 | 2.9 | 2.4 | 2.3 | 1.8 | 2.4 | 1.9 | 2.4 | 1.9 |
| $r_i$ | 0.018 | 0.059 | 0.099 | 0.029 | 0.038 | 0.095 | 0.072 | 0.021 | 0.019 | 0.055 |

| **Inst. 5** ($c = 10$,$k = 6623$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 1.0 | 2.0 | 1.0 | 2.0 | 2.0 | 1.0 | 1.0 | 3.0 | 2.0 | 1.0 |
| $h_i$ | 0.63 | 0.44 | 0.98 | 0.37 | 0.95 | 0.82 | 0.14 | 0.56 | 0.38 | 0.12 |
| $a_i$ | 929.47 | 3734.65 | 6285.2 | 3405.1 | 1995.09 | 1400.92 | 7022.46 | 4053.93 | 4290.27 | 534.52 |
| $b_i$ | 2.8 | 1.7 | 2.7 | 2.2 | 1.9 | 2.0 | 2.2 | 1.6 | 2.0 | 2.5 |
| $r_i$ | 0.056 | 0.02 | 0.063 | 0.083 | 0.018 | 0.06 | 0.043 | 0.085 | 0.03 | 0.072 |

| **Inst. 6** ($c = 10$, $k = 5595$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 2.0 | 4.0 | 4.0 | 4.0 | 1.0 | 3.0 | 4.0 | 2.0 | 3.0 | 2.0 |
| $h_i$ | 0.68 | 0.41 | 0.23 | 0.82 | 0.7 | 0.48 | 0.44 | 0.84 | 0.18 | 0.32 |
| $a_i$ | 2900.67 | 1381.02 | 628.43 | 3293.88 | 3289.64 | 3363.23 | 234.75 | 613.37 | 937.7 | 550.6 |
| $b_i$ | 1.8 | 2.6 | 2.1 | 2.7 | 2.7 | 2.4 | 2.0 | 1.8 | 1.9 | 2.0 |
| $r_i$ | 0.094 | 0.036 | 0.022 | 0.041 | 0.084 | 0.067 | 0.021 | 0.042 | 0.021 | 0.03 |

| **Inst. 7** ($c = 10$, $k = 9725$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 1.0 | 2.0 | 4.0 | 2.0 | 1.0 | 1.0 | 4.0 | 3.0 | 3.0 | 2.0 |
| $h_i$ | 0.24 | 0.23 | 0.94 | 0.47 | 0.75 | 0.5 | 0.39 | 0.48 | 0.58 | 0.18 |
| $a_i$ | 3202.42 | 3777.52 | 344.13 | 842.73 | 2894.51 | 7123.17 | 595.35 | 7025.1 | 178.04 | 1469.85 |
| $b_i$ | 1.8 | 2.1 | 1.7 | 3.0 | 2.0 | 2.5 | 2.8 | 1.8 | 2.3 | 2.2 |
| $r_i$ | 0.069 | 0.067 | 0.088 | 0.053 | 0.011 | 0.081 | 0.035 | 0.079 | 0.012 | 0.059 |

| **Inst. 8** ($c = 10$, $k = 5996$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 4.0 | 2.0 | 3.0 | 4.0 | 2.0 | 1.0 | 1.0 | 4.0 | 2.0 | 3.0 |
| $h_i$ | 0.35 | 0.9 | 0.28 | 0.98 | 0.35 | 0.31 | 0.11 | 0.37 | 0.52 | 0.99 |
| $a_i$ | 1336.24 | 7514.05 | 2548.69 | 2066.2 | 838.94 | 3333.54 | 13355.88 | 1413.39 | 832.28 | 1419.53 |
| $b_i$ | 2.8 | 2.8 | 2.5 | 2.7 | 2.6 | 2.9 | 1.8 | 1.9 | 1.9 | 2.8 |
| $r_i$ | 0.019 | 0.096 | 0.03 | 0.027 | 0.033 | 0.092 | 0.014 | 0.041 | 0.048 | 0.014 |

| **Inst. 9** ($c = 10$, $k = 5505$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_i$ | 2.0 | 4.0 | 1.0 | 4.0 | 4.0 | 2.0 | 2.0 | 1.0 | 4.0 | 1.0 |
| $h_i$ | 0.96 | 0.29 | 0.15 | 0.66 | 0.92 | 0.4 | 0.86 | 0.24 | 0.59 | 0.47 |
| $a_i$ | 2978.69 | 548.3 | 6278.22 | 2198.47 | 3812.5 | 5238.36 | 4070.88 | 10692.41 | 726.1 | 1965.56 |
| $b_i$ | 1.8 | 1.5 | 2.4 | 1.8 | 3.0 | 2.5 | 2.4 | 1.9 | 1.9 | 1.8 |
| $r_i$ | 0.017 | 0.082 | 0.042 | 0.026 | 0.06 | 0.084 | 0.014 | 0.054 | 0.081 | 0.064 |