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Abstract

Preventive maintenance planning is an important problem for the handling of energy production systems with high down time costs. Throughout the last decade different maintenance strategies have been developed and optimized in order to minimize operational and maintenance costs whilst conserving and improving the system reliability and productivity. Preventive maintenance strategies are usually based on the monitoring and the prediction of the system behavior and its deterioration process. However, some industrial systems may be operating under a dynamic environment and/or variable working conditions. In this case both the deterioration and the production processes may not be deterministic and incorporate different types of uncertainties. In this paper, we consider the case of a preventive maintenance strategy for a production system subject to uncertainty. For this system, a decision-making procedure for condition-based maintenance planning is proposed. In order to consider uncertainty in production and deterioration processes, these latter are modeled by non-monotonic stochastic processes. The modeling of deterioration processes by means of jump-diffusion stochastic processes has been proposed in our previous work. In this paper, a decision-making approach for preventive maintenance strategies is proposed. Knowing the remaining useful life of a system, a simulation-based real options analysis is used in order to determine the best date to maintain. Considering a case study of a wind turbine with PHM structure, the decision-making approach is described and tested through an empirical example.

1 Introduction

For industrial systems, the operating and maintenance charges generally present a significant amount of the average cost of production. In order to reduce excessive and unnecessary charges as well as system failure and unavailability, an optimization of the

maintenance strategy and scheduling should be considered. The maintenance strategy has a significant impact on the performance of industrial system (availability, downtime, productivity, cost of production and life cycle cost ...) [10]. In this context, an appropriate maintenance strategy is the key factor for the enhancement of system reliability and profitability. Different strategies have been proposed in literature according to various decision criteria (life-cycle-cost [17], system reliability [22], availability [33]). Among these strategies, corrective maintenance strategy is usually adopted when the impact of system failure does not introduce high financial losses or safety risks. However, time-based and condition-based preventive maintenance strategies are adapted more often when the consequences of system failure are considered to be very costly in regards to safety and finances.

Nowadays, preventive maintenance strategies have become very common and are adopted in different disciplines: for instance power plants and distribution networks, the transport industry, data centers, network devices and food processing plants, etc. These strategies are usually based on the observation and the analysis of the system behavior and its deterioration level. Failure prediction is also found to be a fundamental step in the optimization of predictive maintenance scheduling and implementation. In order to avoid unanticipated failures, many industries, such as aircraft, wind turbines and electronic systems, have started to incorporate prognostics and health management (PHM) techniques. The role of a PHM technique is to provide information on the remaining useful life (RUL) of the system. These techniques also allow to predict the advent of failure giving the operator the opportunity to manage and apply appropriate actions in order to restore the system's health before failure. PHM methods and technologies require the definition of measurable health indicators for different components as well as multiple monitoring and analysis approaches.

In this paper,

- a flexible model named jump-diffusion process is proposed for deterioration and production modelling
- a real option inspired by financial options is proposed to define a preventive maintenance rule
- under given constraint, the maintenance decision making is optimized.

The originality of the paper is due to

- the production model: the use of jump diffusion deterioration process for the first time in stochastic production modelling.
- the maintenance policy: the use of real option for the first time in maintenance planning combined with jump diffusion deterioration process.

The first section of this paper is devoted to the description of sources and types of uncertainties. Afterwards, for the special case of wind turbine system is considered, production and deterioration modeling with a stochastic process. A decision-making approach using option theory is discussed. Finally, an empirical example is given for the case of wind turbine maintenance planning in order to demonstrate the utility of the proposed approach.

2 Production and use of real option for maintenance planning

Failure prediction and prevention is directly related to the analysis of the deterioration mechanism of the system and its critical components. Deterioration process analysis provides major information about the remaining useful life of the system before failure. However, for most of the complex systems nowadays the deterioration process is not deterministic and may include different sources of uncertainty. Moreover, a continuous and direct observation of the deterioration level is usually not feasible. For these systems, the definition and observation of measurable deterioration signals or health indicators became a promising method for deterioration analysis and system's health monitoring. Using available data from health indicators, the evolution of the deterioration mechanism can be modeled and therefore failure time prediction, according to a predefined failure-threshold, can be applied. In this context, finding a proper deterioration model is a fundamental step in the system's health management and maintenance scheduling. In the last decades, the modeling of deterioration processes has become a major concern for system engineers and researchers. In literature, the deterioration process is usually considered as a random phenomenon and should be modeled with probabilistic tools [49]. The deterioration mechanism due to normal operation of the system usually occurs gradually over time in a sequence of positive increments. In this case, the deterioration is considered as a monotonic process. Stochastic processes with monotonous evolution, such as Gamma process and its extensions, are used for the modeling of monotonic deterioration processes [44, 40, 45, 26]. However, non-monotonic deterioration phenomena were also observed and discussed in literature. For instance, non-monotonic deterioration signals have been observed for motor bearings [11], heat exchangers [1], LED light display [3], fatigue data of metals [32] and wear particles in oil [41]. In this case, stochastic processes with non-monotonic evolution, based on Brownian motion, are used, refer to [28, 41]. To define and estimate the remaining useful life of the system or one of its components, based on the deterioration model and observations, plays a major role in maintenance planning and optimization. Non-monotonic diffusion processes (Wiener and UO processes) were particularly used by Valis et al. [41] for the modeling of the occurrences of particles in oil.

The author also presented a brief but rich literature review on RUL estimation methods highlighting the role of this diagnostic on the optimization of preventive maintenance planning.

Moreover, the production process of an industrial system can be altered by different factors such as the demand forecast and internal and external supply processes, etc. Major sources and types of uncertainty that arise in manufacturing contexts are discussed by Graves [14]. In fact, the randomness in production processes is not a new topic. Daughety [8] considers that most of production processes are stochastic by nature. He proposes a stochastic model for production output and cost, while considering the presence of noise in the production process due to machine failures, the variability of labor quality and system imperfections. A thorough literature review of models used for production planning under uncertainty is given by Mula et al. [29]. Among the considered uncertainties, the author presents different research works that apply stochastic models to describe environmental and system uncertainties such as production supply [43], market demand [21], lead times [48] and production capacity [50].

Recently, stochastic programming and optimization methods have been introduced in order to add uncertainty to production planning models. Moreover, probabilistic tools have been used for the modeling of uncertainty in production processes in different fields. For instance, stochastic processes are used to model uncertainties in agricultural production [2], soil production [19], and the electricity market [27]. For the latter, several uncertainty sources of different types are considered: energy production cost [27], electricity prices [7], demand and consumption evolution [46] as well as inflow for hydropower [35] and wind speed distributions for wind power [5]. In all these examples the uncertainty has been modeled by means of stochastic processes.

In this paper, we consider an industrial system with production and deterioration uncertainties. We consider that the overall deterioration mechanism of the system is related to the health condition of different components and depends on several internal and external working conditions. The cumulative damage, due to the random effect of usage and age on the system health condition, is considered to produce a continuous, generally increasing but non-monotonic deterioration process. Therefore, this process can be represented by a Wiener-based stochastic model. In addition to this cumulative damage, if the system is operating under a dynamic or stochastic environment, one should consider the effect of the external variation on the system's health. Extreme external conditions, as well as working under faulty or unhealthy conditions may introduce additional damage to the system and cause sudden and large fluctuations in the deterioration process. The best candidate for such a model is a jump diffusion process, refer to Hanson [18], Kou and Wang [24]. This process has time-dependent Gaussian distributed increments with possible exponential or uniformly distributed random jumps. These models have been

also used in health indicators modeling, see [12, 13]. This deterioration model for the wind energy production system is presented in the following section.

For the wind turbines, preventive maintenance strategies usually require the implementation of condition monitoring and prognostic and health management techniques (PHM). These techniques are used in order to assess the actual reliability and health conditions of the system according to health and deterioration signals measurements. The PHM system allows the prediction of an upcoming failure which gives the operator the appropriate time to plan and preform necessary maintenance actions before (or upon) the expiry of the system's RUL. After the PHM signal, the operator would have several opportunity dates to perform the maintenance. In order to choose the best date to preform maintenance a decision-making tool based on an economic criterion is proposed in this section. For our decision approach, we suggest the use of the "theory of option" essentially derived from the financial domain where uncertainty and risks are particularly important.

In finance, an option is a contract which gives the owner the right, but not the obligation, to buy (Call option) or sell (Put option) an underlying asset at a specified strike price before or upon a specified maturity date. The strike price may be set by reference to the current price at the marketplace, or it may be fixed at a discount or at a premium [34]. The value of the option can be calculated at any time t by an evaluation (or recovery) process based on forecasts made on the price development of the underlying asset given its current price at time t. This calculation, also known as the theory of options, depends, in method and in result, on the chosen model for the representation of the underlying asset price. Autoregressive and stochastic models, including Wiener based models [6] with or without jumps [25] and diffusion models with stochastic volatility [42] are most often used in option evaluation studies. Considering these processes for the modeling of asset prices, various numerical methods for the estimation of options value before and upon the maturity date have been proposed [47, 25]: Monte Carlo simulations, fast Fourier transforms, Laplace transformation and least squares Monte Carlo approach (LSM)...

Although the term "option" is often used to identify a type of contract in the financial field, this term is widely used to describe various applications in the engineering domain or even in daily-life situations. In real life, the term "option" may refer to any optional action or opportunity that is not necessarily related to financial instruments or derivatives. For example, a factory manager may be able, but not compelled, to hire additional employees or buy new equipment in order to improve the production output. However, the factory manager should analyze the profitability of this project in order to take the proper decision.

Today, the use of real option as a decision support tool in non-financial applications is continually increasing. For example, Davis [9] introduced the concept of real options

in order to evaluate the profitability of a gold exploration project and to manage exploration policies. For this evaluation the authors consider geological and economic sources of uncertainty. Considering production and demand uncertainties, Jin et al. [20] uses the theory of option in order to propose a new cost model for scheduling joint production and maintenance actions. In this paper, the authors consider the increasing uncertainty in customer demands which is represented as a stochastic model. In fact, real options have become a recognized valuation, project management and strategic decision paradigm that applies financial option theory to real-life decisions [37]. The analogy between real options and financial options, as well as their significant differences, are highlighted by Haahtela [15]. Although many of real option evaluation procedures are derived from the financial option evaluation methods, these two problems do not necessarily follow the same assumptions. Haahtela [15] suggests that the real value of any method is based on its practical relevance and ability to help practitioners make correct and timely investments, or at least avoid making clearly incorrect decisions. The author concludes that unlike financial options the evaluation procedure of several real options should implement numerical methods such as simulation and lattice methods rather than difficult mathematics. An equivalence between financial option and real maintenance option in prognostics and health management system (PHM) is also given by Haddad et al. [16].

In this paper, we propose the use of real option as a decision-making tool for preventive maintenance planning of a production system subject to uncertainty. Furthermore, different sources of uncertainty may be considered for the modeling of the production process. In this paper, we only consider two sources of uncertainties: (1) the production quality related to the system deterioration level and (2) environment state or external process.

3 Deterioration and production modeling

Wind turbine operation and deterioration processes are generally subject to multiple factors and dynamic conditions. Stochastic processes present a natural choice for modeling of the production and deterioration uncertainties.

3.1 Production process

The output power produced by a wind turbine is usually affected by several factors, including uncontrollable natural factors such as wind speed and direction. The capacity and power coefficients of the turbine also affect the wind power production. Both of these coefficients are not constant and may depend on wind speed and turbine's age [39]. An example of daily returns for a wind turbine during the final period of deterioration before failure is given by Niknam [30] in Figure 1. Considering all these factors a stochastic

model will be proposed for the representation of the instant energy production. In the model, the influence of wind speed fluctuation as well as the power and capacity factors will be considered. We also suppose that these two factors are directly related to the deterioration level of the turbine.

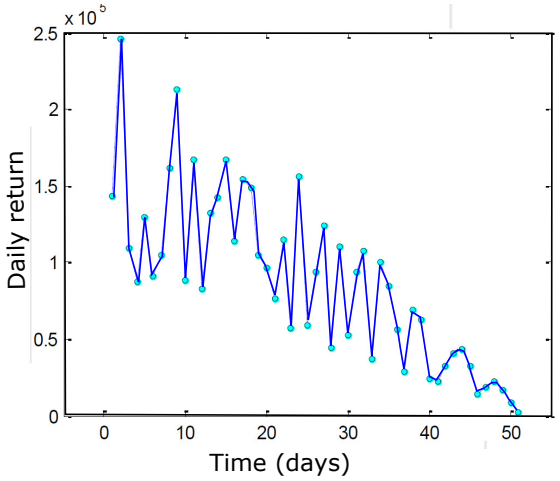

Figure 1: Example on daily return of a wind turbine in a final deterioration state [30].

The system deterioration level at time t is denoted by $D(t)$ and the environment state or external process is modeled by a proper continuous stochastic process $V(t)$. The production output process is denoted by $P(t)$ and is detailed as follows.

Let P_0 be the nominal production output by unit of time. The instantaneous production output process $P(t)$ is proposed to be modeled as a function of the nominal production added to both deterioration and environment fluctuations effects. Considering also that the production output is equal to zero after the system failure, the production process $P(t)$ can be finally written as follows:

$$P(t) = \begin{cases} P_0 t + V(t) + g(D(t)), & \text{if } t < T_{fail} \\ 0, & \text{if } t \geq T_{fail} \end{cases} \tag{1}$$

where $g(D(t))$ is a function of the deterioration process $D(t)$ representing the effect of deterioration of the system on the production output; T_{fail} is the effective date of failure and $V(t)$ is a stochastic process representing the effect of external factors and environment uncertainties on the production output. For our case study $g(D(t))$ is chosen as a linear function as follows:

$$g(D(t)) = I_{d/p} D(t) \tag{2}$$

where $I_{d/p} < 0$ is a constant coefficient describing the effect of deterioration on production.

3.2 Deterioration process

In this step, we consider a single deterioration process for the overall system. This process should represent the deterioration level of the system's multiple components. We also consider that this process can be affected by several factors (external and internal) as well as the instantaneous working conditions and stress. Note that a sudden evolution of these conditions may introduce additional damage to system components. Based on these assumptions the deterioration process is proposed to be modeled by the following jump-diffusion process (independent and time-dependent Gaussian distributed increments with possible random jumps).

Let $D(t)$ be the corresponding process representing the deterioration of the system. The dynamics of $D(t)$ can be described by the following stochastic differential equation (SDE):

$$dD(t) = D(t)[\mu_D dt + \sigma_d dW(t) + \xi_J dN(t)], \tag{3}$$

starting at $D(0) = D_0$, where μ_D is the drift coefficient associated with the diffusion, σ_D is the diffusive volatility, $W(t)$ is a one-dimensional stochastic Wiener process, ξ_J is the Poisson jump amplitude and $N(t)$ is the standard Poisson process with joint mean and variance $\mathbb{E}[N(t)] = \text{var}[N(t)] = \lambda t$ which marks the arrival times of jumps.

If $D(t)$ is defined as a non-negative process, as in the case of a deterioration process, a logarithmic form of equation (3) can be deduced using the stochastic chain rule [34],

$$d\ln D(t) = \mu_{ld} dt + \sigma_d dW(t) + J dN(t) \tag{4}$$

$$\Leftrightarrow \quad D_L(t) = \ln D(t) = \mu_{ld} t + \sigma_d W(t) + \sum_{i=0}^{N(t)} J_i \tag{5}$$

$$\Leftrightarrow \quad D(t) = e^{[D_L(t)]} = e^{\left(\mu_{ld} t + \sigma_d W(t) + \sum_{i=0}^{N(t)} J_i\right)} \tag{6}$$

where $\mu_{lnD} = \mu_D - 0.5\sigma_d^2$ is called the log-diffusive drift and $J = \ln(\xi_J + 1)$ is a random variable representing the increment jump's amplitude. The variable J can be modeled according to various distributions (e.g. normal, uniform and double exponential distribution). Using available data of one or multiple health indicators, the calibration of the jump-diffusion process can be performed according to a weighted least squares function (χ^2 fit) applied to the increment distribution [18, 12, 13].

Among others, jump-diffusion model with double exponentially distributed jump amplitude is particularly interesting for deterioration modeling. Due to the memoryless property of exponential distribution, this model presents a rare case in which an analytic solution of the first passage times τ_L for a given threshold L by the process $D_L(t)$ can be derived. The explicit solution for the Laplace transform of the distribution of the first passage time τ_L is given by Kou and Wang [24]. The distribution of the first passage time

is particularly indispensable for the estimation of remaining useful life of a system (RUL). The estimated value plays a fundamental role for decision-making regarding preventive maintenance actions.

The influence of environmental conditions can be modeled by covariates impacting the deterioration process parameters. For instance, it is possible to consider a Markov chain suggested by [51] to model covariates. For wind farms the variation in environmental conditions and wind speed seasonality could justify the use of a Markov chain. However, in order to simplify the description of the proposed approach and the corresponding numerical calculation, we first consider that the deterioration model is limited to a diffusion process with double-exponential jumps and constant parameters. In the last section of this paper, the case of Markov switching environment is also considered for the empirical example.

3.3 Prognosis

Prognosis and lifetime prediction have been largely developed in theory and industry. The quantity of interest is often the Remaining Useful Lifetime (RUL) and is estimated based on the degradation measures. The main purpose of degradation modeling is to allow the prediction of failure and future behavior of the system. For deteriorating systems, the failure occurs when the health indicator exceeds a given threshold called failure threshold. The probability of failure is calculated as the probability of the first hitting time. In this framework, the Remaining Useful Lifetime (RUL) can be defined as follows:

$$\begin{aligned} t_{RUL}(t) &= \inf\{h \geq 0; D(t+h) \geq S_p | D(t) < S_p, \Psi_t\} \\ &= \inf\{h \geq 0; D(t+h) - D(t) \geq S_p - d | D(t) = d < S_p, \Psi_t\} \end{aligned} \tag{7}$$

where Ψ_t is the set of information available at time t.

In practice, according to the deterioration level at time t denoted by $D(t)$, the probability distribution function of the first hitting time of the level L by the process is calculated. The mean value of this distribution is considered as the RUL at this time and denoted by $T_{RUL}(t) = \mathbb{E}(t_{RUL}(t))$.

Considering a given indicator and its corresponding calibrated model, the estimation of the first hitting time of the predefined threshold can be done by two methods. The first method is to directly use the probability distribution function of the first hitting time and derive confidence intervals for the failure time. The second method is to generate a large number of trajectories of this model by Monte Carlo simulations and give an empirical distribution of the first hitting time. The first method can be implemented in a very short time but it requires the close form of the distribution. The second method is more time consuming but it does not require a knowledge of the hitting time distribution. In

the case of jump-diffusion processes with exponential jumps, the exact distribution of the hitting time distribution is available via its Laplace transform, refer to [24]. The numerical calculation of the Laplace transform and its inversion are also very time consuming. In this paper a mixture of the two methods is used to obtain the first hitting time distribution. Monte Carlo simulations are used to approximate the Laplace transform and then derive the distribution.

4 Use of real option for maintenance optimization

As we have mentioned, in finance, an option is a contract which gives the owner the right, but not the obligation, to buy (Call option) or sell (Put option) an underlying asset at a specified strike price before or upon a specified maturity date. At the maturity date T, the value (payoff) of an option can be easily calculated. Let's take for example the case of a Call option. This option gives its holder the right to purchase an asset A for a fixed price K at a fixed maturity date T. At the maturity date T, the payoff of the option can be defined as the maximum possible gain following the exercise of this option: "buying the asset A at a fixed price K". This value can be formulated as follows:

$$C_{val} = \max(0, Y(T) - K) = (Y(T) - K)^+ \tag{8}$$

where $Y(T)$ is the price of the asset A at time T.

The value of a put option, which represents the maximum possible gain following the exercise of the option "selling the asset A at a fixed price K at the time T", can be calculated in a similar way:

$$P_{val} = \max(0, K - Y(T)) = (K - Y(T))^+ \tag{9}$$

In the following section we propose the use of real option as a decision making tool for preventive maintenance planning of production system subject to uncertainty.

4.1 "Wait-to-maintain" option for wind turbine maintenance

Wind energy industry has experienced impressive growth over the past years. Wind energy has great potential to lessen our dependence on traditional resources like oil and gas and to do it with less damage to the environment. However, onshore and offshore wind farms are facing numerous challenges that could inhibit their competitiveness in the energy market. Wind turbines are exposed to variable and difficult weather conditions, including high or severe wind, lightning, arctic cold, hail and snow. These external conditions, as well as the high load that a wind turbine is exposed to, may lead to intense mechanical stress and a high risk of failure [36]. Note that the availability of wind turbines, as well as their life-cycle-cost, have a direct impact on the profitability of a wind farm and consequently on

the investment trends in this domain. In this context, failure prediction and maintenance optimization have now become important issues in wind industry. Ultimately, operating and maintenance cost of wind turbines form a large amount and a significant part of the average cost of energy production [10]. An example of maintenance costs for offshore wind farm is given by Besnard [4] for the Horns Rev wind farm in Denmark. For this farm, and because of certain early failures, maintenance costs amounted to almost 40% of the life-cycle-cost of turbines. Among the different activities, corrective maintenance, component replacement and major renovations present the main contribution in the total cost of maintenance.

In addition to the corrective maintenance cost, early failures generally cause a considerable downtime of the production system and may introduce additional financial costs such as penalties. Penalty costs usually arise from the obligation to buy, or to alternatively reproduce the energy no longer supplied by the faulty turbine(s) [38]. Note that in the case of an offshore park, the downtime period of a turbine could last longer important because of transport and logistics constraints. Considering these constraints, and in order to avoid severe financial consequences of early failure, the development and adoption of an appropriate maintenance strategy is required.

In the following sections, we will represent an option-based decision-making approach for preventive maintenance planning of wind turbines. An empirical example is given in order to illustrate the proposed approach. The wind turbine is considered as a single-unit system in terms of system monitoring and maintenance activities.

In this paper, the wind turbine maintenance problem under consideration is directly inspired by the problems encountered by wind farm maintenance operators. Due to their inaccessibility and harsh environmental conditions, the systems are not continuously inspected, but are equipped with sensors giving information on the health indicators. The maintenance operations may be carried out at relaying predefined dates due to logistics and asset management constraints. However, in this framework, it is possible that the planned operation dates arrive either too early or after a failure. This paper aims to reduce this risk resulting from non-optimal decision making.

The following assumptions are considered:

- **The system is equipped with a PHM monitoring system. When the deterioration level of the system reaches a predefined alarm threshold, a warning signal is produced. This signal particularly includes an estimation of the remaining useful life of the turbine T_{RUL}.**
- **After a PHM warning signal preventive maintenance activities should be planned and carried out in order to prevent the predicted failure.**
- **The system is considered to be in failure mode when the deterioration**

level reaches a predefined failure threshold for the first time. The effective date of failure is denoted by T_{Fail}. After this date, the production process is considered to be equal to zero and corrective maintenance should be applied at the earliest opportunity date in order to restore the system's health.

- **Predictive and corrective maintenance can only be performed at specific and predefined dates (maintenance opportunity dates). A minimum delay of $\Delta_i T$ is needed in order to plan and prepare the maintenance activity. In this study, we assume that maintenance opportunities are equally separated. In practice, the interval between maintenance opportunity dates ($\Delta_i T$) may vary with the season, logistics conditions and the estimation of current and future load charges.**

After the reception of a failure warning signal at $t = t_0$, the operator has to choose the best time to perform maintenance. Three main timings can be differentiated:

1. **the current time t_0; preventive maintenance is performed. However, this option is impossible in practice because a delay is necessary to plan a maintenance operation. Thus, preventive maintenance is planned and performed at the nearest opportunity date.**
2. **the maturity time $t_f = t_0 + T_{RUL}$; preventive maintenance is applied after the expiry of the estimated residual life,**
3. **an opportunity date t_k between t_0 and t_f; preventive maintenance is planned for the date t_k where $t_k = t_0 + k\Delta_i T$ with $k \geq n$ and $n\Delta_i T$ is the minimum requested delay for planning and preparing the maintenance activity due to asset management and logistics constraints.**

The last two choices give the operator the chance to wait before applying any maintenance activity. This option, called the "wait-to-maintain" option may introduce extra benefits because of the additional running time of the system. However, the risk of failure and the cost of preventive maintenance can also increase because of the accumulative damage during this operation. An ideal compromise for the maintenance timing problem would be to find the proper date to maintain by considering the minimization of preventive and corrective maintenance costs and penalties and the maximization of the operational useful time of the system.

4.2 The "wait-to-maintain" option valuation

Compared to an immediate maintenance at t_0, the importance of the "Wait-to-Maintain" option lies in the cumulative return that can be collected from additional operation of

the system before maintenance. For the valuation of this option, an estimation of two essential quantities is needed:

- $R_{cum}(t_k)$: the cumulative return produced by the additional operation of the system between t_0 and t_k;
- $C_{sup}(t_k)$: the augmentation of maintenance costs between t_0 and t_k. This quantity includes the augmentation of the preventive maintenance cost because of additional damage to the system between t_0 and t_k as well as the augmentation of the failure risk and costs. This latter includes the cost of corrective maintenance and replacement activities and the accumulation of penalties during the downtime of the system.

Considering these two quantities, the value of "Wait-to-Maintain" option for a planned maintenance at time t_k can be given as follow:

$$O(t_k) = \max(0, R_{cum}(t_k) - C_{sup}(t_k)) = (R_{cum}(t_k) - C_{sup}(t_k))^+ \tag{10}$$

The optimal maintenance date planned at t_0 will be defined as follows:

$$T_{op}^{mp} = \underset{t_k \in \{t_0+\Delta_i, t_0+2\Delta_i, \cdots, T_{RUL}\}}{\operatorname{argmax}} \mathbb{E}[O(t_k)] = \underset{t_k \in \{t_0+\Delta_i, t_0+2\Delta_i, \cdots, T_{RUL}\}}{\operatorname{argmax}} \mathbb{E}[(R_{cum}(t_k) - C_{sup}(t_k))^+ | \Psi_0] \tag{11}$$

where Ψ_0 is the information available at time t_0.

Cumulative return $R_{cum}(t_k)$ The cumulative return of the system production between t_0 and t_k can be defined as the cumulative production of the system during this period, multiplied by the price of energy per unit (C_{prix}):

$$R_{cum}(t_k) = C_{prix}. \int_{t_0}^{t_k} P(t)dt \tag{12}$$

C_{prix} is supposed to be constant and $P(t)$ is the random production process represented by equation (1). The expectation of the cumulative return of production between t_0 and t_k can be calculated as follow:

$$\mathbb{E}[R_{cum}(t_k)|\Psi_0] = C_{prix}. \quad \mathbb{E}[\textstyle\int_{t_0}^{t_k} P(t)dt | \Psi_0] \tag{13}$$

where Ψ_0 is the information set at t_0 including particularly the estimation of the remaining useful life T_{RUL} and the deterioration level $D(t_0)$.

The expectation in (13) can be developed according to two possible cases:

- $T_{Fail} > t_k$: no failure occurs before t_k. The turbine is operating normally and the instantaneous production $P(t)$ is represented by equation (1) with $t < T_{fail}$ over the whole $[t_0, t_k]$ interval .

- $T_{Fail} \leq t_k$: a failure occurs at time T_{Fail} between t_0 and t_k. The turbine is operating normally on the interval $[t_0, T_{fail}[$ and the instantaneous production on this interval is represented by the equation (1) with $t < T_{fail}$. For the rest of the time, i.e. for the interval $[T_{Fail} t_k]$ the system is down and the production output can not be profitable. In this case we have $P(t) = 0$.

Considering these two possibilities, equation (13) can be developed as follows:

$$\mathbb{E}[\int_{t_0}^{t_k} P(t)dt|\Psi_0] = \mathbb{E}[\int_{t_0}^{t_k \wedge T_{Fail}} (P_0 + V_v(t) + I_{d/p}.D(t)dt|\Psi_0]$$
$$= \mathbb{E}[(t_k \wedge T_{Fail} - t_0)]P_0 + \mathbb{E}[\int_{t_0}^{t_k \wedge T_{Fail}} V_v(t)dt|\Psi_0] + I_{d/p}.\mathbb{E}[\int_{t_0}^{t_k \wedge T_{Fail}} D(t)dt|\Psi_0] \quad (14)$$

In this equation $D(t)$ is a stochastic jump-diffusion process and T_{Fail} a random variable representing the effective failure time. The distribution of T_{Fail} can be deduced analytically using its Laplace transform given by Kou [23]. Integrals of type $A_t^{(v)} = \int_{t_0}^{t_k} exp(\mu_{ld} t + \sigma_d W(t))dt$ with t_k constant have been devoloped in literature for financial applications [31]. For these applications analytic solutions of $A_t^{(v)}$ distribution function are derived. However, the integration of a Wiener-based process with additional jump and/or non-deterministic limits have not been explored. For the empirical example in section 5, numerical simulations are used in order to calculate the expectation value of equation (14).

Augmentation of maintenance costs The total increase in maintenance costs $C_{sup}(t_k)$ can be divided into two main parts according to failure possibilities: 1) if $T_{Fail} > t_k$: the augmentation of preventive maintenance cost C_{sup}^{pr} due to accumulative damage of the system; 2) if $t_0 < T_{Fail} \leq t_k$: the augmentation of corrective maintenance cost C_{sup}^{cr} due to early failure.

Therefore $C_{sup}(t_k)$ can be calculated as follow:

$$\mathbb{E}[C_{sup}(t_k)|\Psi_0] = \mathbb{P}(T_{Fail} > t_k)\mathbb{E}[C_{sup}^{pr}(t_k)|\Psi_0, T_{Fail} > t_k]$$
$$+ \mathbb{P}(T_{Fail} \leq t_k)\mathbb{E}[C_{sup}^{cr}(t_k)|\Psi_0, T_{Fail} \leq t_k] \quad (15)$$

Considering that the cost of preventive maintenance is proportional to the deteriora-

tion level with a constant factor $C_{d/m}$, we can write:

$$\begin{aligned}\mathbb{E}[C^{pr}_{sup}(t_k)|\Psi_0, T_{Fail} > t_k] &= C_{d/m}.\mathbb{E}[D(t_k) - D(t_0)|\Psi_0, T_{Fail} > t_k] \\ &= C_{d/m}.\mathbb{E}[D(t_k)|\Psi_0, T_{Fail} > t_k] - C_{d/m}.D(t_0) \\ &= C_{d/m}.\mathbb{E}[D(t_0) + \int_{t_0}^{t_k} dD(t)|\Psi_0, T_{Fail} > t_k] - C_{d/m}.D(t_0) \\ &= C_{d/m}.\mathbb{E}[\int_{t_0}^{t_k} dD(t)|\Psi_0, T_{Fail} > t_k] \end{aligned} \tag{16}$$

On the other hand, $\mathbb{E}[C^{cr}_{sup}(t_k)|\Psi_0, t_0 < T_{Fail} \leq t_k]$ represents the expected cost to afford in case of early failure. If a failure occurs before the scheduled preventive maintenance, corrective maintenance is imposed. The cost of corrective maintenance includes the cost of system repair and components replacement C_{rep}. C_{rep} is considered to be constant and significantly higher than the cost of preventive maintenance. This cost also includes eventual penalties. Penalty is usually applied because of the unavailability of the turbine and the production loss. Thus, penalty cost can be expressed in function of the system downtime $t_k - T_{Fail}$:

$$\begin{aligned}\mathbb{E}[C^{cr}_{sup}(t_k)] &= C_{rep} + I_{pen}.\mathbb{E}[(t_k) - T_{Fail}|\Psi_0, T_{Fail} \leq t_k] \\ &= C_{rep} + I_{pen}.(t_k) - \mathbb{E}[T_{Fail}|\Psi_0, T_{Fail} \leq t_k] \end{aligned} \tag{17}$$

where I_{pen} is a penalty factor per downtime unit (one day). This factor is assumed to be constant.

5 Empirical Example

In order to demonstrate the decision-making approach, an empirical example is presented in this section.

The system and model parameters of this example are basically inspired from simulated and real data examples of literature. $V_v(t)$ is considered as a Brownian motion with zero drift and a volatility equal to 0.5. The considered parameters for the deterioration process as well as the values of different factors used for our example are given in Table 1. Note that the deterioration process is chosen to vary between 0% and 100%. The warning signal threshold is fixed at 65% and a failure is considered to occur when the deterioration level reaches the 100% level.

Using these parameters, deterioration and production trajectories can be simulated and their related failure time, cumulative returns and maintenance cost can be calculated. Examples of deterioration $D(t)$, wind related process $V_v(t)$, production process $P(t)$ paths and cumulative return paths are represented in Figure 2 .

P_0	C_{prix}	C_{rep}	$I_{d/p}$	$C_{d/m}$	$D(t)$ model parameter						$V_v(t_0)$
MWh	euro/MWh	10^3 euro	MW	10^3 euro	μ_{ld}	σ_d	λ_J	μ_1	μ_2	p_1	-
60	13.5	600	-0.12	3	0.05	0.1	20	0.05	0.1	0.2	10

Table 1: Model and system parameters for empirical example.

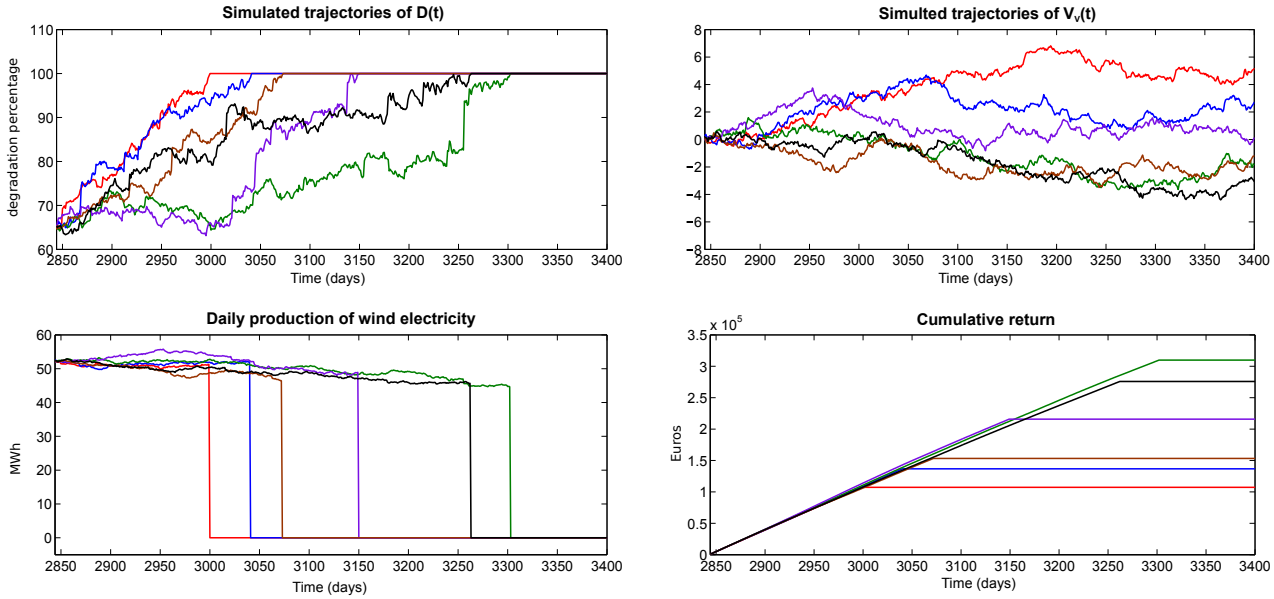

Figure 2: Examples on simulated trajectories of production, deterioration and wind speed related processes.

Figure 3 presents examples of simulated deterioration trajectories from a double exponential jump diffusion process. The warning threshold is set to 65% of the failure threshold. Figure 4 presents histograms of t_0 (or T_{Alarm}) and T_{Fail} being the first passage time to the alarm threshold S_{Alarm} and failure threshold S_{Fail} respectively. One can notice a shift of 220 days between the two histograms, which means that, on average, the turbine is operational 220 days after the warning alarm. This hypothesis is confirmed by Figure 5, where the histogram of the estimated remaining useful lifetime (RUL) from the simulated deterioration trajectories is depicted. In this case, the average RUL calculated at t_0 is $\mathbb{E}[T_{RUL}] = 219.5$ days and its variance is $Var[T_{RUL}] = 1150$ on 50000 trajectories.

Consider one trajectory where the warning signal is produced at $t_0 = 2850$ days with deterioration level $D(t_0) = 65$ and an estimated $T_{RUL} = 210$ days. The maintenance optimization of this case could be performed. For this optimization the maximization of $O(t_k)$ is required.

Consider the red trajectory in Figure 3 as the real trajectory of the deterioration indicator. At $t = 0$ the turbine is new. After 2844 days, the deterioration level reaches the warning threshold and warning alarm is sent to the operator and at this time the remaining

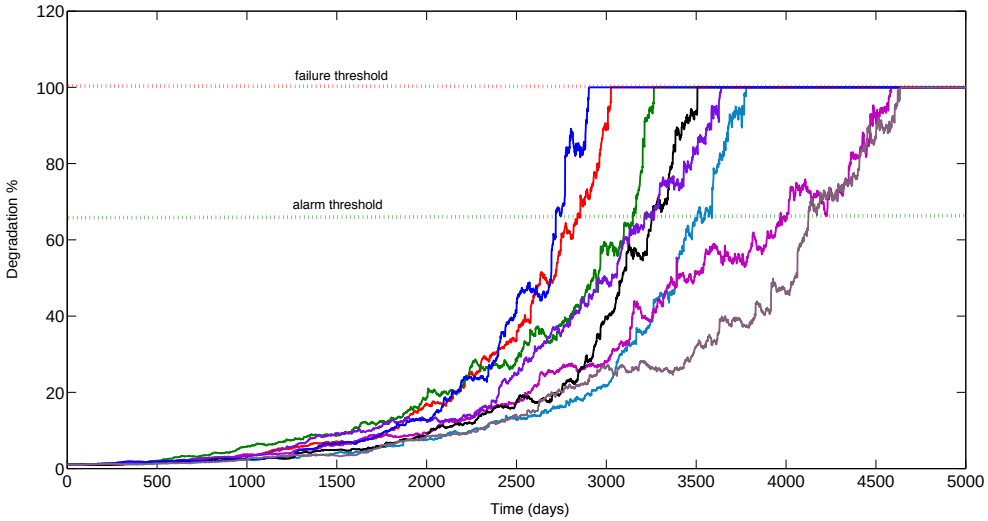

Figure 3: Simulated deterioration trajectories.

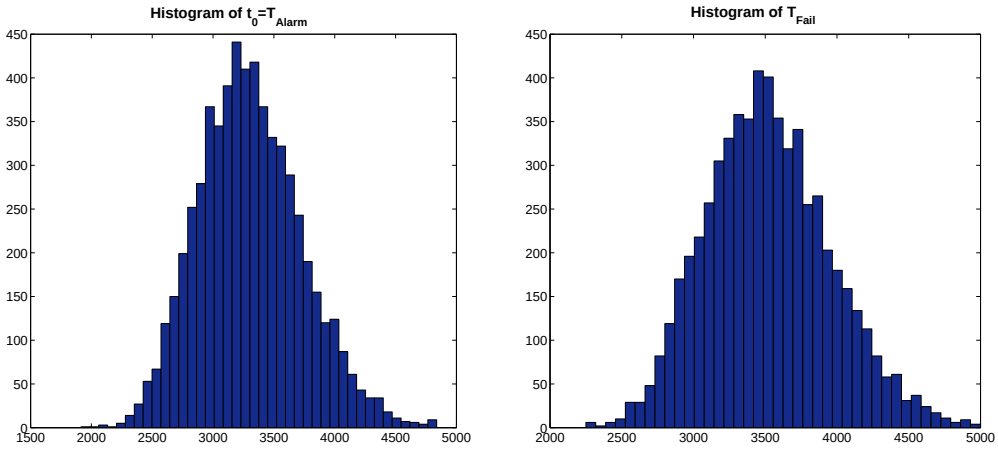

Figure 4: Histograms of T_{Alarm} and T_{Fail}.

useful lifetime average $E(T_{RUL})$ is estimated to 220 days. Due to logistic, calendary and environmental constraints, the operator can carry out the preventive maintenance operations at predefined times. To plan an optimal date for the maintenance operation, the operator will apply for a "wait-to-maintain " option evaluation for every possible date before the remaining useful lifetime average.

Using Monte Carlo simulations, for different opportunistic maintenance intervals $\Delta_i T$ and maintenance dates, the "wait-to-maintain" value $O(t_k)$ can be estimated. Considering the opportunistic maintenance interval $\Delta_i T = 10$ days, the expectation of cumulative return and maintenance cost augmentation with respect to maintenance dates are depicted in Figure 6 and the "wait-to-maintain" value $O(t_k)$ is highlighted.

The estimation of the "Wait-to-Maintain" option values (O) for mainte-

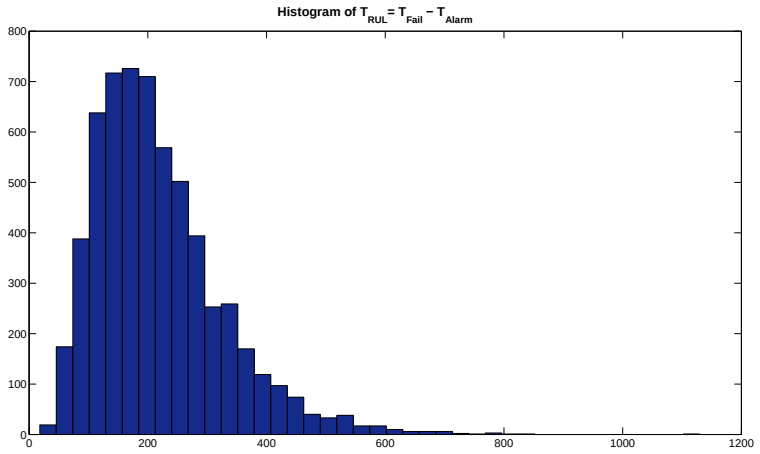

Figure 5: Histogram of T_{RUL} at time t_0.

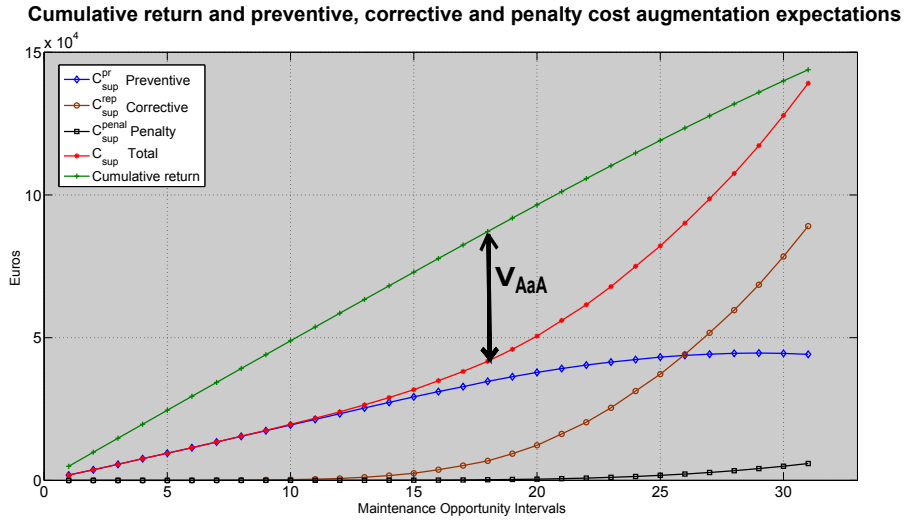

Figure 6: Expected values of cumulative return and the augmentation of maintenance costs between $[t_0, t_f]$ ($\Delta_i T = 10$ days).

nance opportunity dates between t_0 and $t_f = t_0 + T_{RUL}$ with different opportunity intervals ($\Delta_i T \in \{10, 30, 50\}$) are represented in Figure 7. These values represent the expectation of the total additional gain accumulated from the system operation until dates $t_k = t_0 + k\Delta_i T$ with $k \geq 1$. From this figure we can deduce that the optimal date to perform preventive maintenance is $T_{op} = 2984$ for $\Delta_i T = 10$. If the preventive maintenance is scheduled for this date, a cumulative gain of $49,930$ euros is expected. Compared to a preventive maintenance at the earliest opportunity date after warning, the "Wait-to-Maintain" option can lead, on average, to additional benefits of $40,580$ euros.

Next, we go further in our empirical study and consider changing working and environmental conditions. To this aim, a regime switching model is used. In this model, the

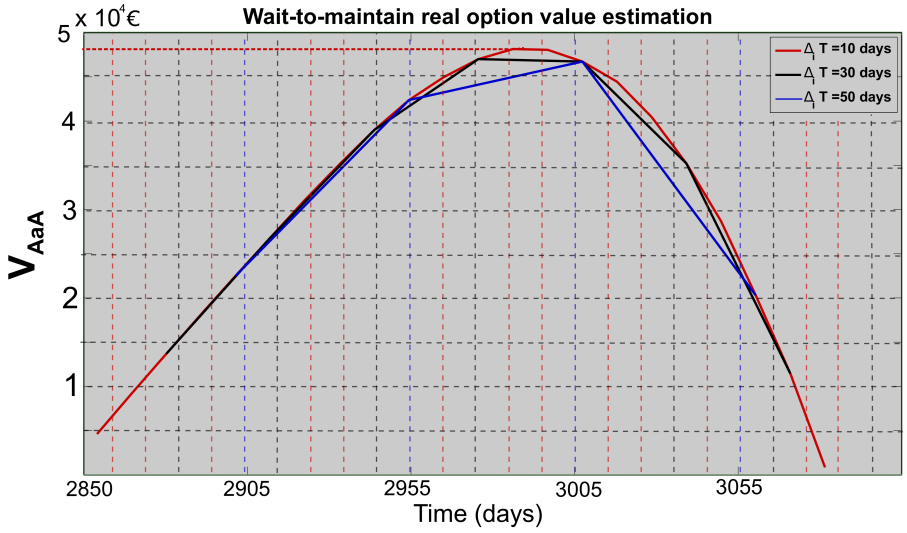

Figure 7: O: Additional gain between t_0 et t_f.

parameters of deterioration and production processes are influenced by an external discrete variable called covariate denoted by Ω representing the working regime. We consider at this stage three regimes: 1) normal regime: where the deterioration and production processes follow the same models as in the first simulation. 2) positive regime: where working conditions and environment states are considered to be advantageous for the production, and the deterioration process is considered to be slower than normal and 3) negative regime: working conditions and environment states are considered to be disadvantageous for the production and the deterioration process is considered to be faster than normal. For each regime the corresponding sets of parameters are fixed for $V_v(t)$ and $D(t)$ processes.

The change of regime (working conditions) can be random or deterministic. We first consider a random covariate modelled by a three state time homogeneous Markov chain. Afterwards, a deterministic but seasonal covariate is considered. In Figure 8 the best time to maintain is obtained by considering the covariate Ω evolving as a Markov chain. In order to be sensible and represent real working condition changes, the Markov chain is supposed to switch very slowly from one state to another. Figure 9 corresponds to the case where the covariate Ω has a deterministic behaviour. In other words, environment conditions can be correctly estimated for the next year with insignificant error (seasonal variation). Although, the corrective maintenance costs are different in the two cases of changing working conditions, the 'wait-to-maintain" values are identical. This result is due to the relatively slow variations of the Markov chain and also that the results are based on mean values.

In comparison with Figures 6 and 7 we can notice in Figures 8 and 9 that the variation of the working conditions impacts the total cost, the production evolution and the

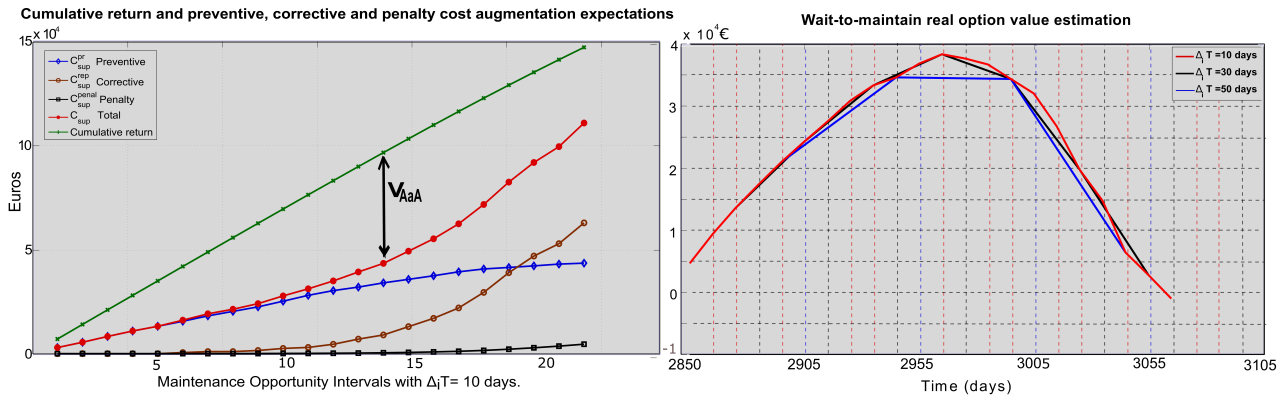

Figure 8: Simulation results considering changing working conditions modelled by a Markov chain: Expected values of cumulative return and the augmentation of maintenance costs between $[t_0, t_f]$ ($\Delta_i T = 10$ days) (Left). O: Additional gain gain in $[t_0, t_f]$ (Right).

estimated value of 'wait-to-maintain" option. Thus, the variation of this estimated value with respect to time strongly depends on the information about environment and working conditions evolution. However, the estimation of the best time to maintain is relatively similar in the three cases ($T_{op} = 2964$ with changing working conditions and $T_{op} = 2984$ for the unchanged working conditions). This result is intuitively expected as the overall mean and variance of the deterioration and production processes evolution are chosen to be similar in the three cases. Nevertheless, precise information about environment and working conditions evolution helps to develop more realistic models and estimate correct value of the maintenance option at the optimal maintenance time.

5.1 Sensitivity analysis to the alarm threshold

Now, for the system under consideration, the impact of the warning threshold on the maintenance policy is studied. or different values of the warning threshold, the real profit and the expected profit until the optimal maintenance date are given in Table 2. One can notice that as the warning threshold increases the warning date occurs later and the maintenance date is planned accordingly. The optimal maintenance date and expected total profit don't change substantially with the warning threshold. The real and expected profits remain stable and the maintenance decision rule is not really impacted.

In order to confirm the obtained results of the sensitivity to the warning threshold, the same sensitivity analysis is carried out on a hundred other indicators. The results for one of these indicators are given in Table 3 depicting matching behavior. Sometimes the expected profit associated to a high warning threshold can be very interesting but in reality, the late maintenance planning can be costly. To find a balance between risk and cost, a warning threshold between 65% and 80% seems reasonable.

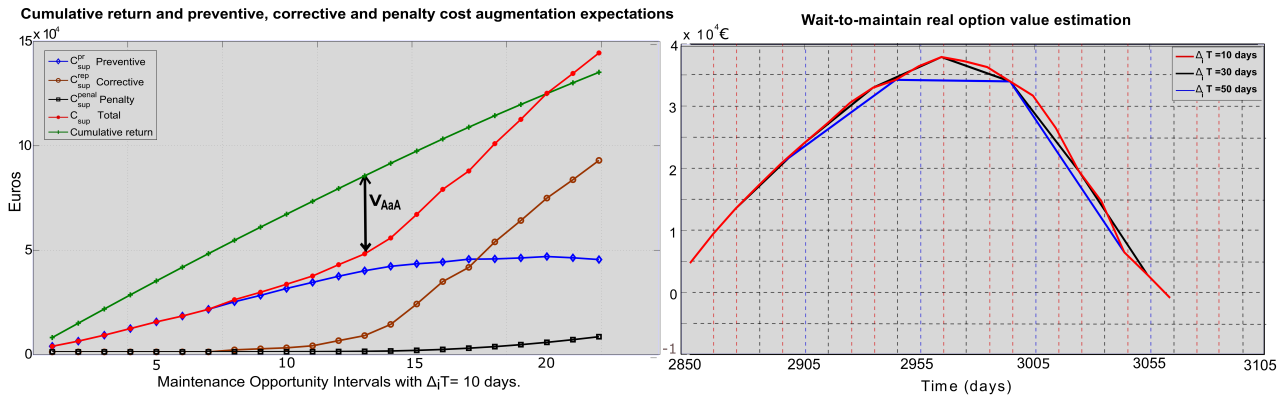

Figure 9: Simulation results considering changing working conditions modelled by a deterministic switching model: Expected values of cumulative return and the augmentation of maintenance costs between $[t_0, t_f]$ ($\Delta_i T = 10$ days) (Left) . O: Additional gain in $[t_0, t_f]$ (Right).

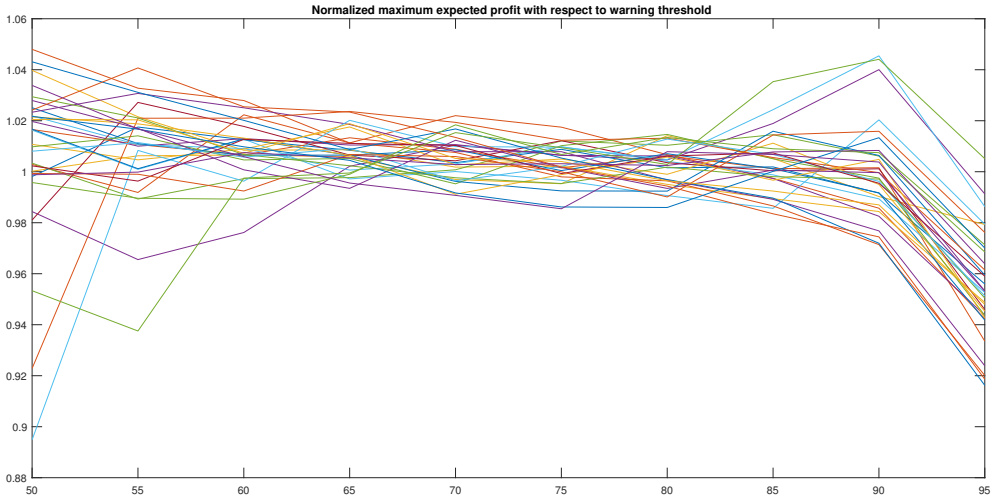

Figure 10: Sensitivity of 12 trajectories of profit with respect to the warning threshold

In Figure 10, the sensitivity of 12 trajectories of profit with respect to the warning threshold are depicted. One can notice that the profit is not very sensitive to the alarm threshold when the latter is between 50% and 90% of the failure threshold. Therefore, maintenance scheduling based on sensible choice of warning threshold leads to an efficient decision rule, even if the threshold is not optimized.

One can notice in the sensitivity analysis that the choice of the warning threshold has no substantial impact on the total profit. For each warning threshold a suitable optimisation procedure can lead to optimal maintenance planning. The crossing time of the warning threshold is random. At this crossing time, the future possible dates of main-

Warning threshold	Warning date	Maintenance date	Expected total profit until optimal maintenance date	Real total profit until maintenance date
50	2736	3056	2133714,754	2119004,707
55	2755	2995	2097161,285	2067959,746
60	2857	3057	2124783,817	2118994,073
65	2858	2980	2092776,526	2073365,025
70	2917	3007	2095321,089	2078473,155
75	2952	3022	2091216,871	2090706,954
80	2969	2999	2076766,234	2071431,938
85	2987	3007	2065785,289	2078473,155
90	3169	3179	2152815,117	2155408,429
95	3216	3226	2120552,826	2166490,539

Table 2: The impact of the warning threshold on the maintenance decision rule and profit.

tenance and different scenarios are considered and accordingly the optimal maintenance date is calculated. The profit optimisation procedure for each warning threshold at the crossing time is only valid for this realization of crossing time. Since the optimisation at each crossing time is very time consuming, and the results of total profit until maintenance time are not very sensitive to the threshold variation, the global optimisation is not considered.

Conclusion

In this paper we have proposed a new approach for maintenance decision-making based on the theory of option. The main objective of this approach is to find the best time to perform preventive maintenance of a given system. This system is considered to incorporate production and deterioration uncertainties. Stochastic jump-diffusion model is proposed to model the random evolution of deterioration process as well as production output. We focus on the case study of wind turbines. The turbine is considered to be equipped with a PHM system. When the PHM detects a system anomaly a warning signal is produced. This signal includes an estimation of the turbine RUL and its deterioration level. We consider that maintenance actions are only permitted at specific dates. To choose the best date between different opportunities, the decision makers have to evaluate the benefit of a real option which characterizes the ability to wait before maintenance. The evaluation of this option can be performed for each date using Monte Carlo simulations. The option value $O(t_k)$ represents the gain reported by the additional operation of the turbine until t_k considering failure possibility. The optimal date to maintain can be found by doing a maximization of $O(t_k)$ with respect to t_k. Empirical examples of this approach are

Warning threshold	Warning date	Maintenance date	Expected total profit until optimal maintenance date	Real total profit until maintenance date
50	2875	3175	2063344,047	2024075,143
55	2904	3144	2035016,678	2019913,761
60	2955	3155	2025925,78	2025236,628
65	3020	3160	2025913,265	2023418,42
70	3087	3187	2030220,625	2016788,787
75	3106	3166	2013720,82	2024994,316
80	3181	3221	2027049,405	2014349,624
85	3201	3221	2015214,206	2014349,624
90	3235	3255	1996125,033	2002589,754
95	3252	3272	1895696,066	2012628,448

Table 3: The impact of the warning threshold on the maintenance decision rule and profit for indicator 2.

presented.

Acknowledgment

This work was supported by the Regional Council of Champagne-Ardenne (RCCA) and the European Regional Development Fund (ERDF) in the framework of RISK PERFOM project.

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